MEM380/MEM800: App Autonomous Robots II

Assignment 9

Due: Monday 3/11 11:59pm EST

1. (5 pts) Let X_1 and X_2 denote two normally distributed random variables, *i.e.* $X_1 \sim N(\mu_1, \sigma_1)$ and $X_2 \sim N(\mu_2, \sigma_2)$. The probability density function for a 1-D normally distributed random variable is given by:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

show that

$$p(X_1)p(X_2) \sim N(\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\mu_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\mu_2, \frac{1}{\sigma_1^{-2} + \sigma_2^{-2}})$$

2. (5 pts) Let $X \sim N(\mu, \sigma)$ and $Y = aX + b$ where $a, b \in \mathbb{R}$. Show that $Y \sim N(a\mu + b, a^2\sigma^2)$.

3. (5 pts) Problem 2(a) in Chapter 2.8 of *Probabilistic Robotics* by Thrun *et al.*

4. (5 pts) Problem 2(b) in Chapter 2.8 of *Probabilistic Robotics* by Thrun *et al.*

5. (5 pts) Problem 2(f) in Chapter 2.8 of *Probabilistic Robotics* by Thrun *et al.*

6. (5 pts) Problem 3(a) in Chapter 2.8 of *Probabilistic Robotics* by Thrun *et al.* Hint: You can use the table provided to generate a state transition graph and use this result in the prediction step.

7. (15 pts) Generate and simulate a Kalman filter to estimate the state variables of the following system:

$$\begin{aligned} x_t &= \begin{bmatrix} -0.4 & 0.2 \\ -0.2 & -0.4 \end{bmatrix} x_{t-1} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t + v_t \\ y_t &= [1\ 0]\, x_t + w_t \end{aligned}$$

where the noise term v_t has zero mean and covariance of $R = [0.03,\ 0; 0,\ 0.09]$ and the noise term w_t has zero mean and variance of $Q = 0.25$. Assume an input $u_t = sin(t)$ over a period of $t \in [0, 100]$ and $x_0 = [0\ 0]^T$. Use a Kalman filter to estimate the state of the system. Plot the actual states, the estimated states, and the measurement.