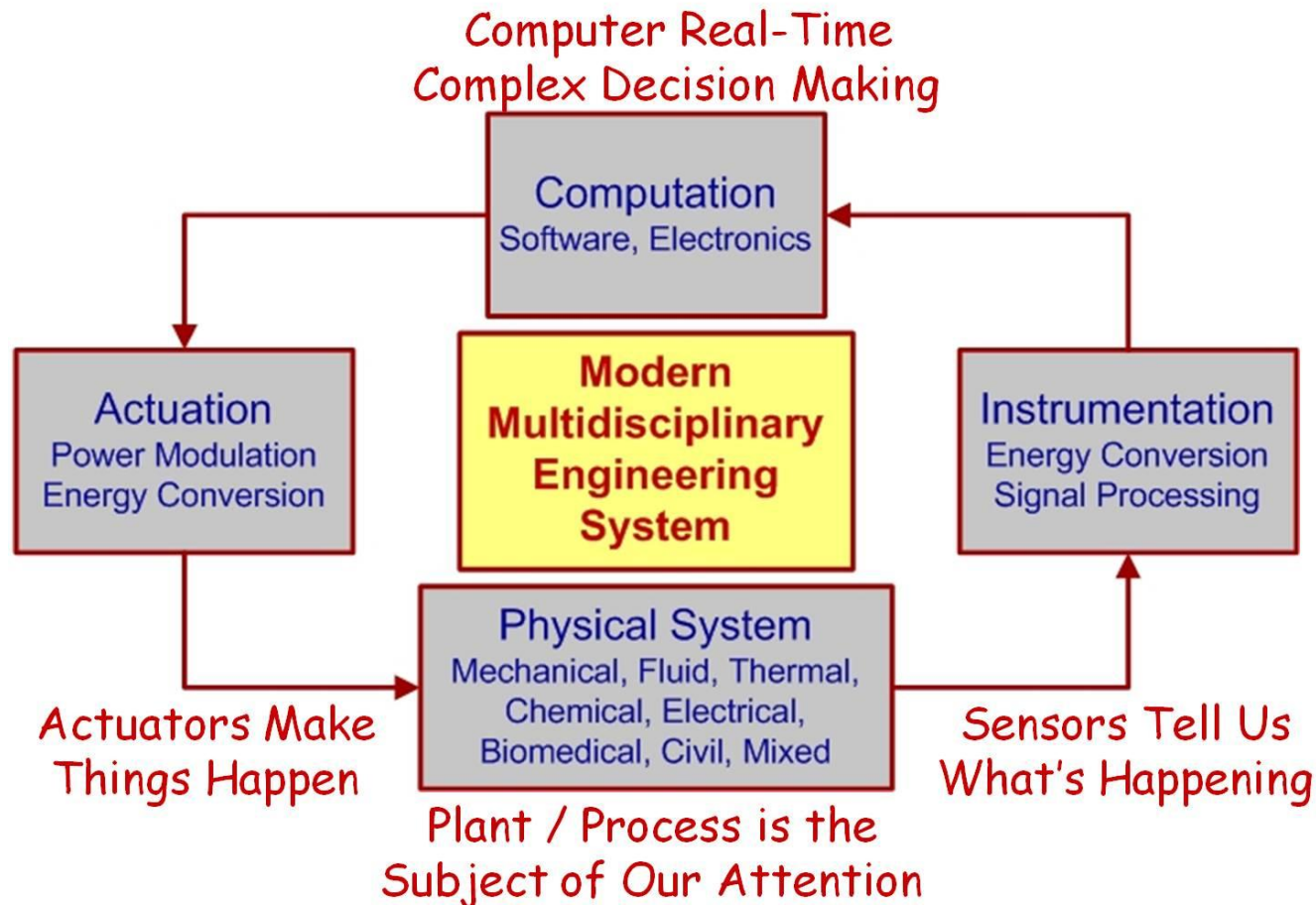


Electromechanical Engineering Systems Mid-Semester Case Study



Course Mid-Semester Case Study

- This integrating experience is to take an existing physical system (electrical first-order system here, but it could be any physical system) and have it meet desired dynamic performance specifications.

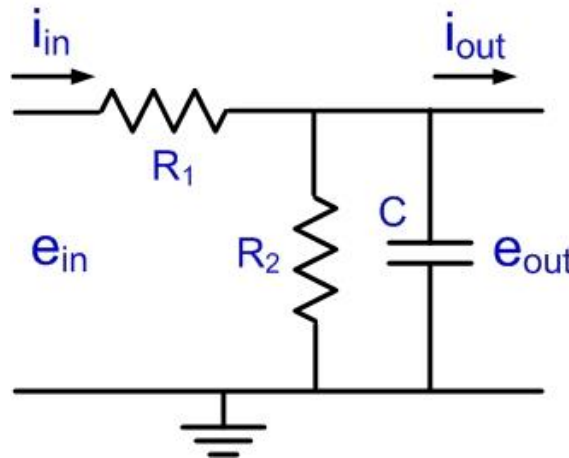
$$t_{r_{10\%-90\%}} \leq 0.015 \text{ sec}$$

$$M_p \leq 25\%$$

$$t_{s_{1\%}} \leq 0.09 \text{ sec}$$

$$\text{Control Effort} \leq 13 \text{ V}$$

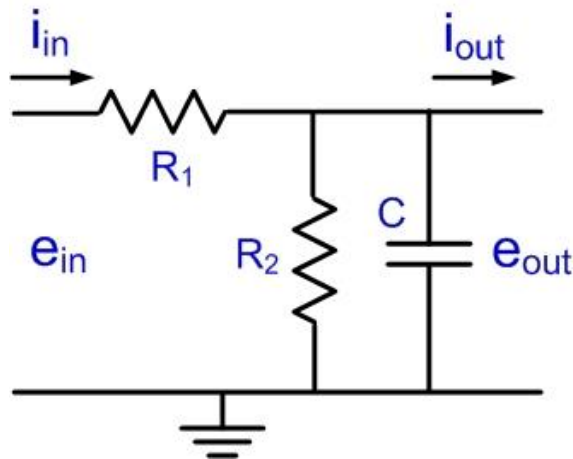
$$\text{SS Error} = 0$$



Electrical
Physical
System

Process

- Can the system alone (i.e., open loop) meet the performance specifications, i.e., unit step response with desired rise time, overshoot, and settling time?



$$R1 = 100 \text{ K}\Omega$$

$$R2 = 100 \text{ K}\Omega$$

$$C = 1 \text{ }\mu\text{F}$$

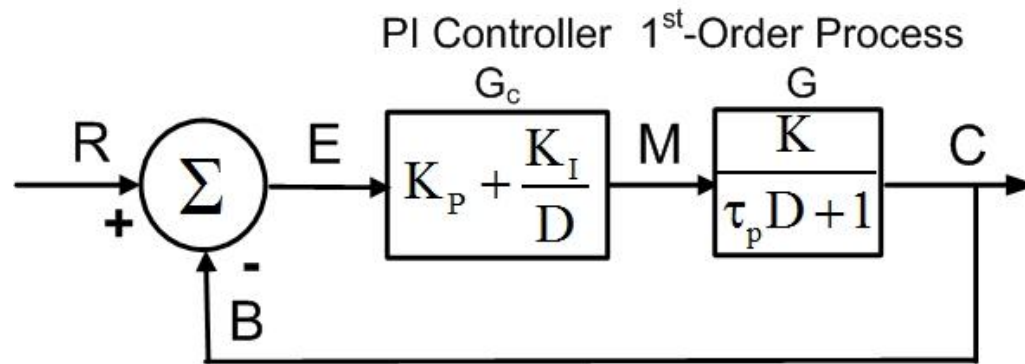
Physical System

- Make simplifying assumptions (e.g., pure and ideal resistors and capacitor, no loading) and create the **physical model**.
- Apply KVL and KCL, along with component constitutive equations, to obtain the **mathematical model**.

$$\frac{e_o}{e_i} = \frac{\frac{R_2}{R_1 + R_2}}{\frac{R_1 R_2}{R_1 + R_2} CD + 1} = \frac{K}{\tau_p D + 1} = \frac{0.5}{0.05D + 1}$$

1st-Order

- **Analysis** (use hand calculation or MatLab analysis) shows that the system time constant is too large and operating open-loop **cannot** meet the performance specifications. What to do?
- Change the physical system! Here we assume that the physical system cannot be changed.
- Apply **closed-loop (feedback) control** to obtain the desired response and use the PI (proportional-integral) controller.



90% of the
controllers used
in the world are
PI Controllers!

$$\frac{C}{R} = \frac{G_c G}{1 + G_c G} = \frac{\frac{KK_I}{\tau_p} \left(\frac{K_P}{K_I} D + 1 \right)}{D^2 + \frac{KK_P + 1}{\tau_p} D + \frac{KK_I}{\tau_p}} = \frac{\omega_n^2 (\tau D + 1)}{D^2 + 2\zeta\omega_n D + \omega_n^2}$$

- Comparison of the actual transfer function with the standard-form transfer function gives the following relationships:

$$\tau = \frac{K_p}{K_i} \quad \omega_n^2 = \frac{KK_i}{\tau_p} \quad \zeta = \frac{1 + K_p K}{2\sqrt{\tau_p K_i K}}$$

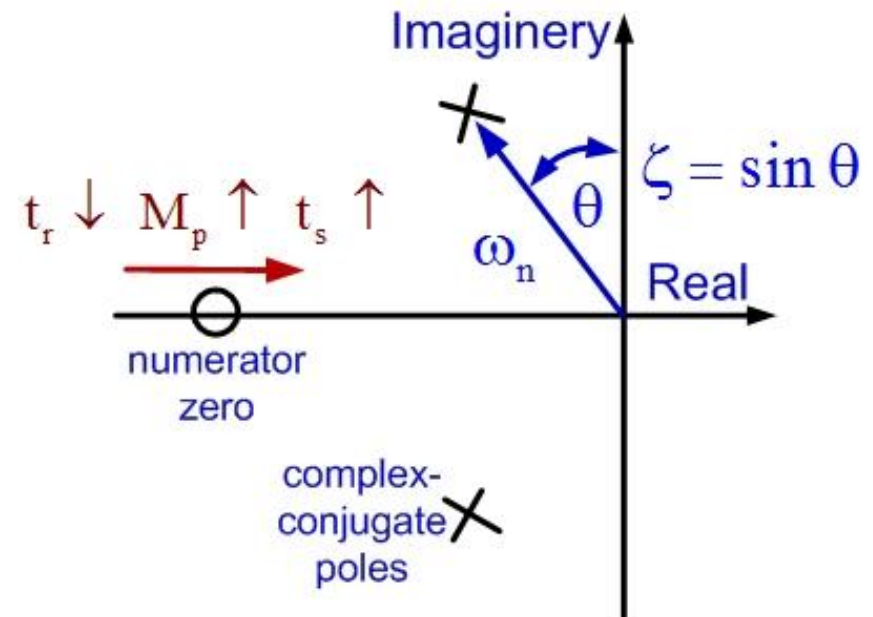
- We now have relationships between the control gains, K_p and K_i , and the dynamic performance indicators for a pure second-order dynamic system, ω_n and ζ .

$$K_i = \frac{\tau_p \omega_n^2}{K} \quad K_p = \frac{1}{K} \left[2\zeta \sqrt{\tau_p K_i K} - 1 \right]$$

$$t_{r_{10\%-90\%}} \approx \frac{1.8}{\omega_n} \quad M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \quad t_{s_{1\%}} \approx \frac{4.6}{\zeta \omega_n}$$

- But Wait! This is not a pure 2nd-order dynamic system. There are numerator dynamics – the numerator has a 1st-order term! Not to worry – why?
- We know that for a pure 2nd-order dynamic system, with the damping ratio ζ between 0 and 1 (typical of most operating engineering systems), the roots of the differential-equation characteristic equation are complex conjugates (indicated by an **x**).

The numerator dynamics, i.e., the 1st-order term, has a root $-1/\tau$ called a zero, indicated with a **o**. As the zero moves along the real axis closer to the pole locations, system dynamic behavior is affected as shown. Take this effect into account in the design!



- Choose $\omega_n = 118$ and $\zeta = 0.64 \rightarrow K_i = 1392$ and $K_p = 13.1$
- The predicted performance values for a pure 2nd-order system are:

$$t_{r_{10\%-90\%}} \approx \frac{1.8}{\omega_n} = 0.015 \quad M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = .073 \quad t_{s_{1\%}} \approx \frac{4.6}{\zeta\omega_n} = .061$$

Note the effect of the zero:

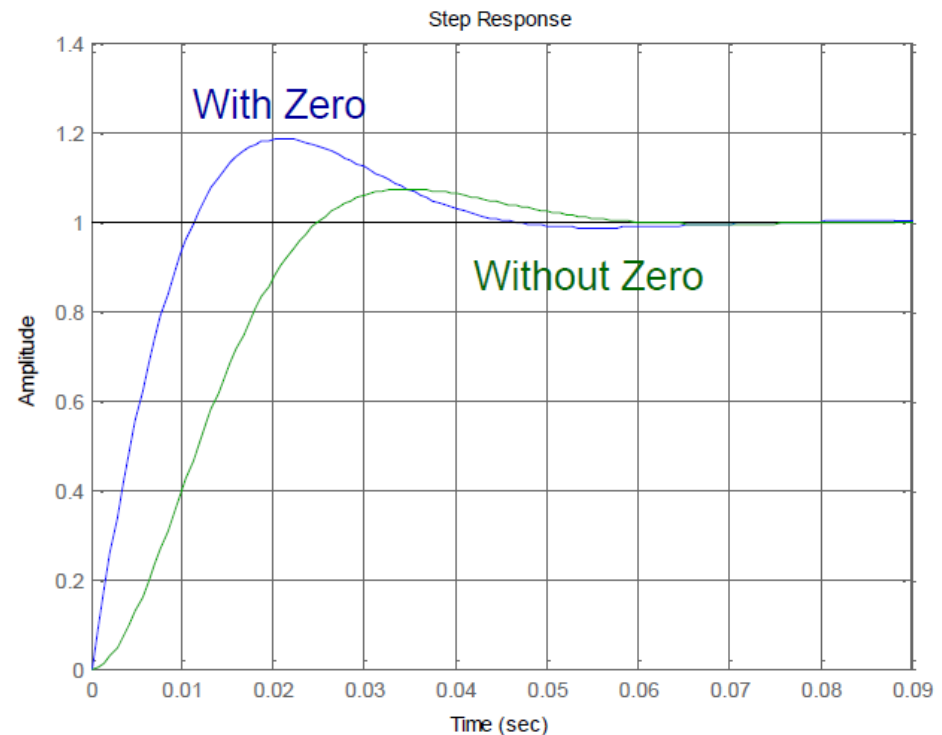
$t_r \downarrow$, $M_p \uparrow$, and $t_s \uparrow$

pole locations: $-75.5 \pm 90.7i$

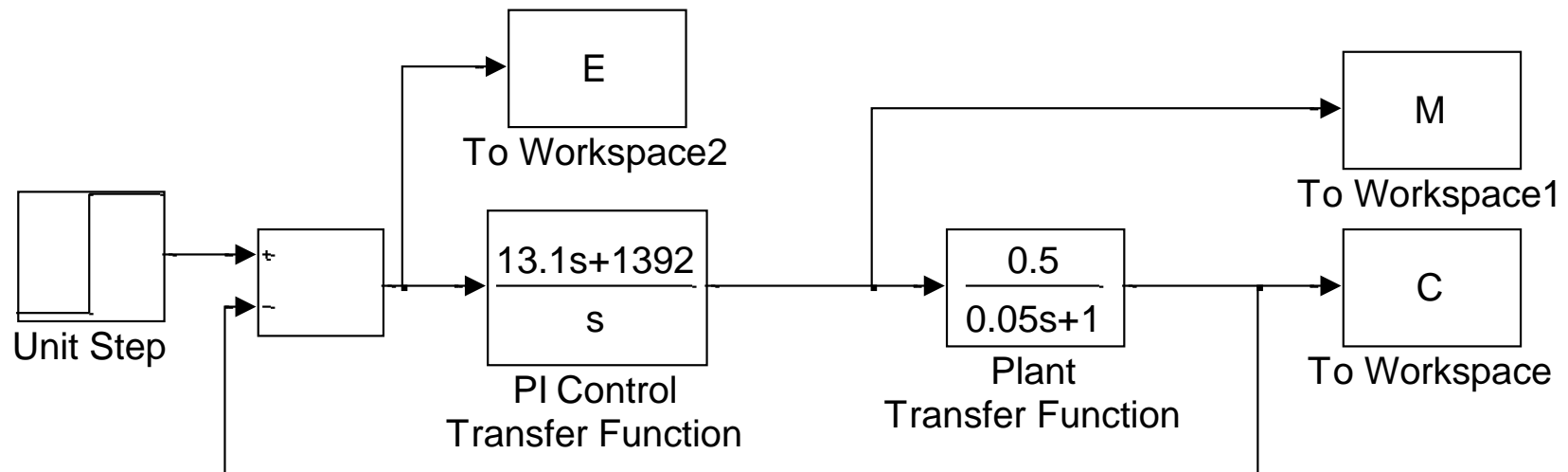
zero location: -106.3

System Performance

$t_r = .0086$ $M_p = .185$ $t_s = .063$



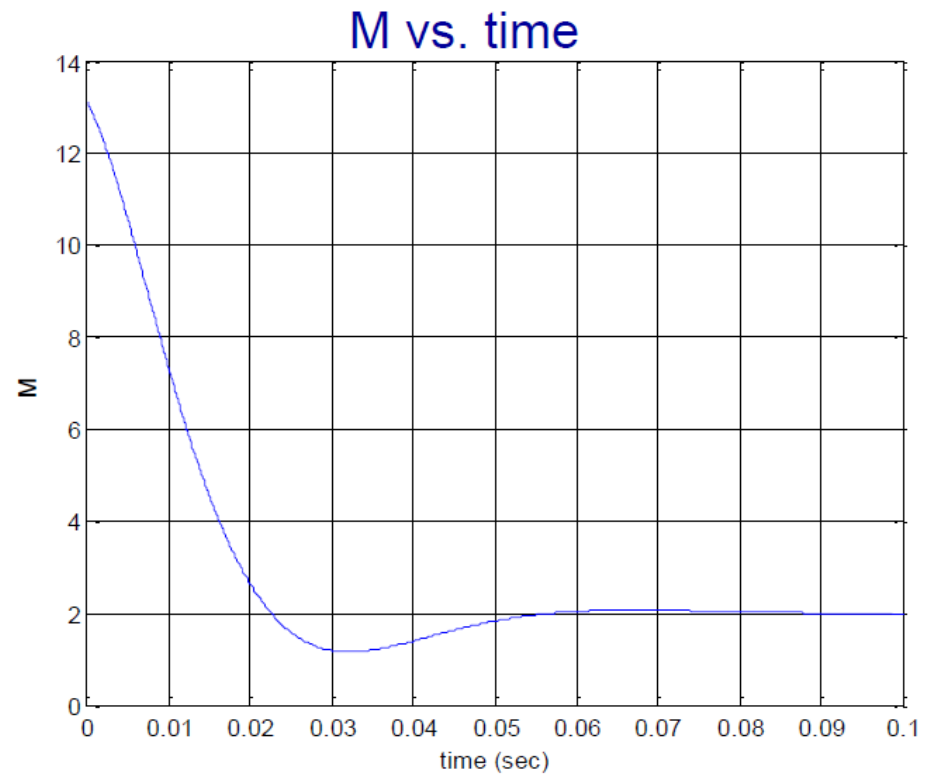
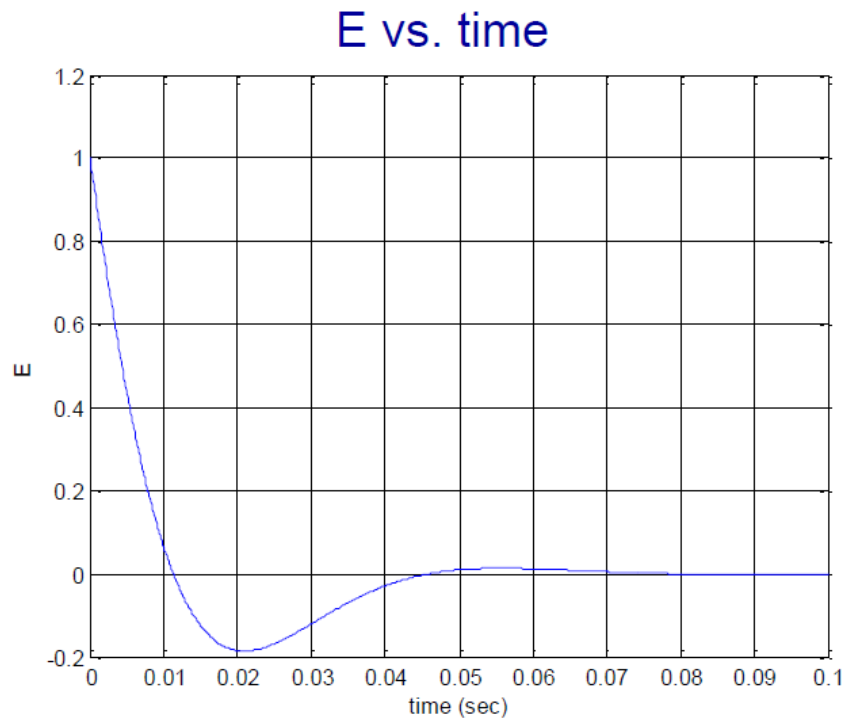
- Our design meets the performance specifications. But what about the control effort? We are going to first implement our design with analog op-amps and we know that the maximum output of an op-amp is about 13 volts when powered by ± 15 V. We use Simulink to check the control effort.



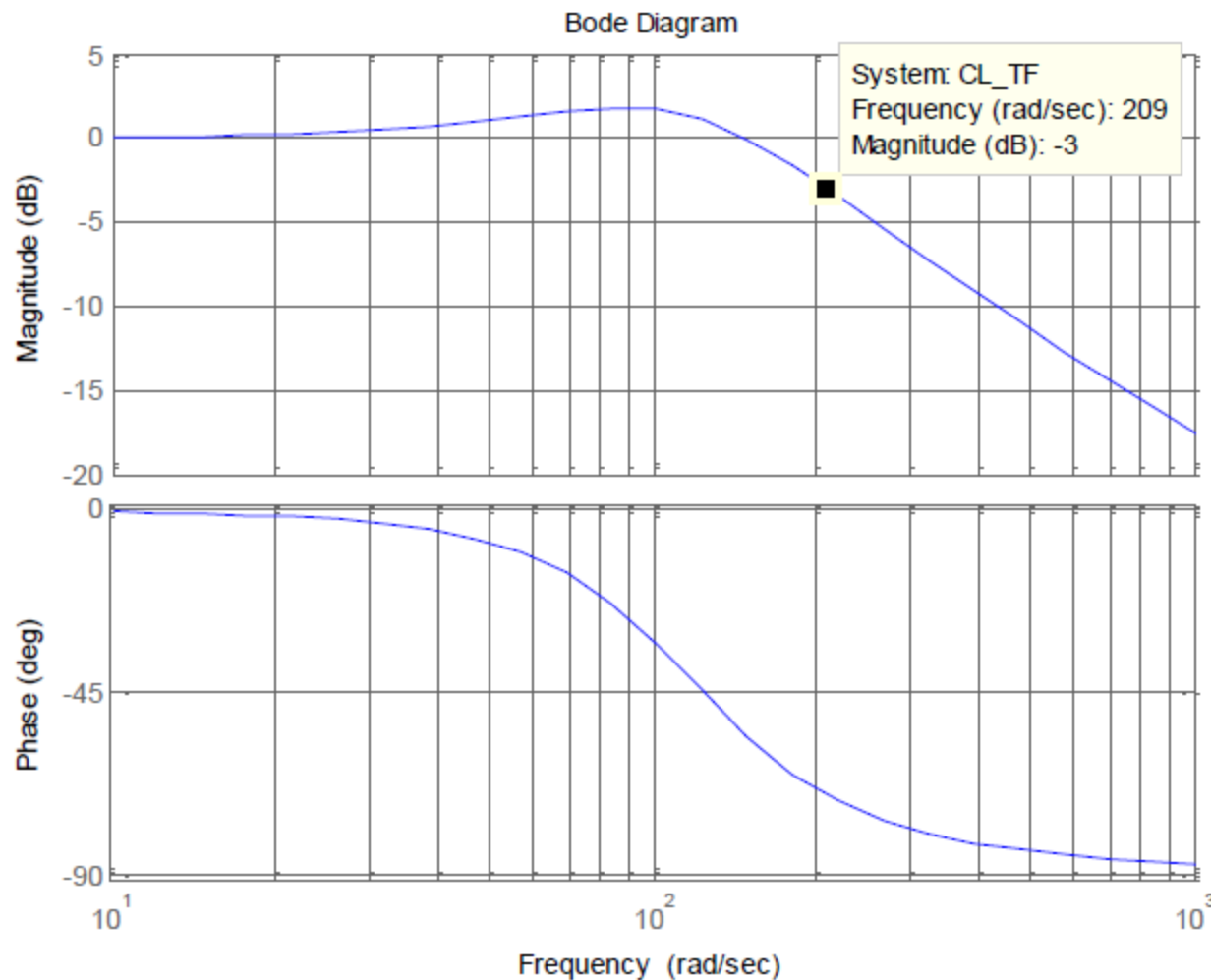
PI Control of a First-Order Plant

MatLab Simulink Block Diagram

Control Effort
 $M < 13$ volts



Error Signal



Bandwidth
33.3 Hz

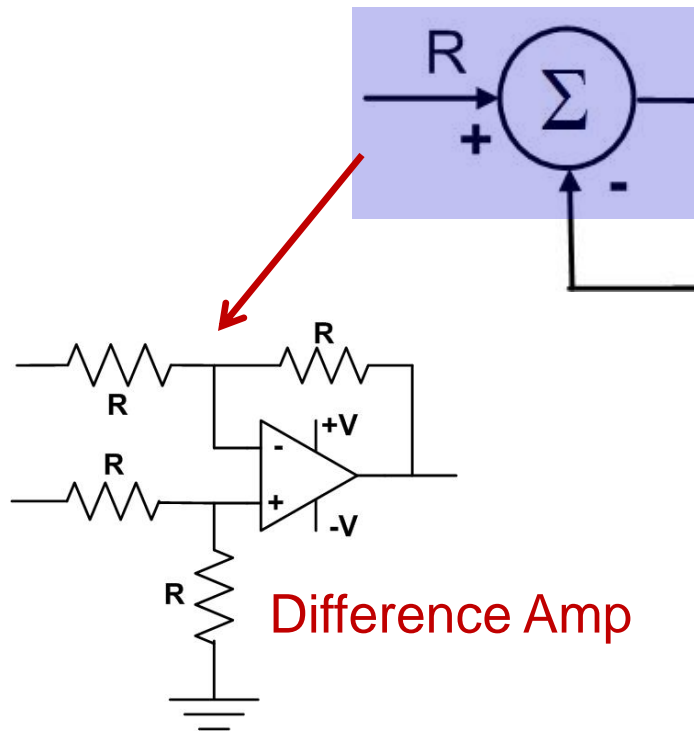
Analytical Closed-Loop Bode Plot

Note: Frequency Response will be studied and used more extensively in the 2nd half of the course.

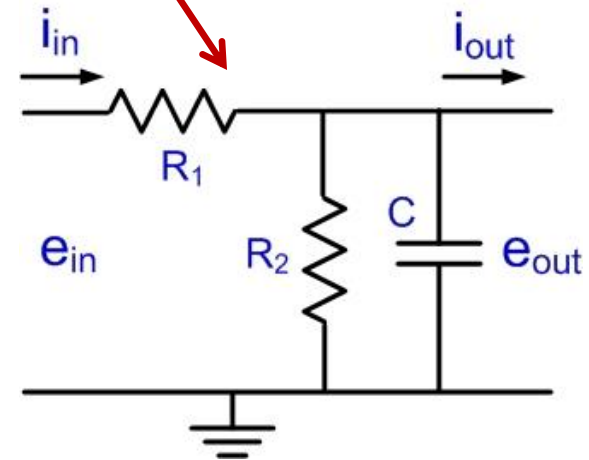
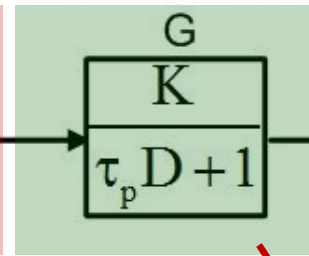
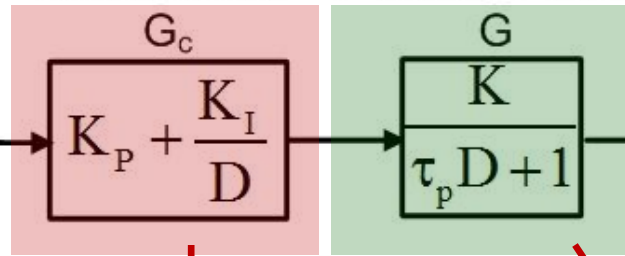
Feedback Control

PI Controller 1st-Order Process

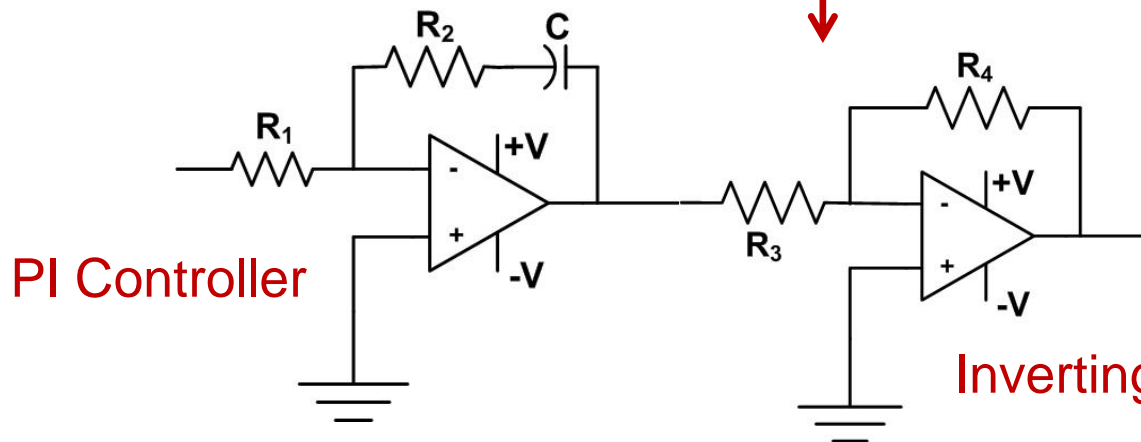
Now
Let's
Build It!



Difference Amp



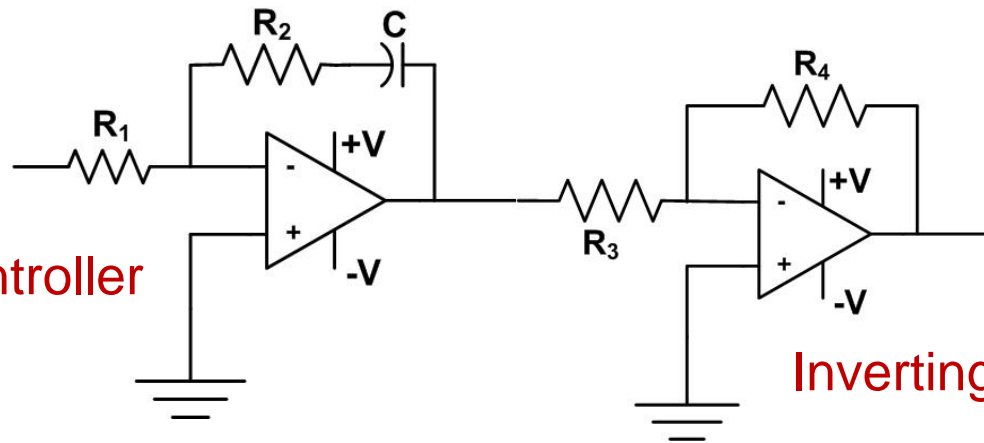
Plant



PI Controller

Inverting Amp

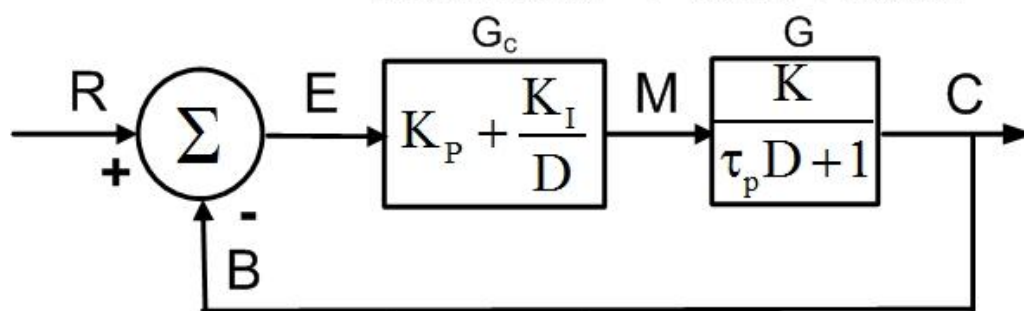
PI Controller



Inverting Amp

$$\frac{e_o}{e_i} = \frac{-\left(R_2 + \frac{1}{C_D}\right)}{R_1} \left(-\frac{R_4}{R_3}\right) = \left(\frac{R_4}{R_3 R_1 C}\right) \left(\frac{R_2 C D + 1}{D}\right)$$

PI Controller 1st-Order Process



$$\frac{M}{E} = K_p + \frac{K_i}{D} = K_i \left(\frac{\frac{K_p}{K_i} D + 1}{D} \right)$$

$$K_i = \frac{R_4}{R_3 R_1 C} \quad K_p = \frac{R_2 R_4}{R_3 R_1}$$

$$\begin{aligned} R_1 &= 1\text{K}\Omega & R_3 &= 1.3\text{K}\Omega & C &= 1.0\mu\text{F} \\ R_2 &= 9.1\text{K}\Omega & R_4 &= 1.8\text{K}\Omega \end{aligned}$$

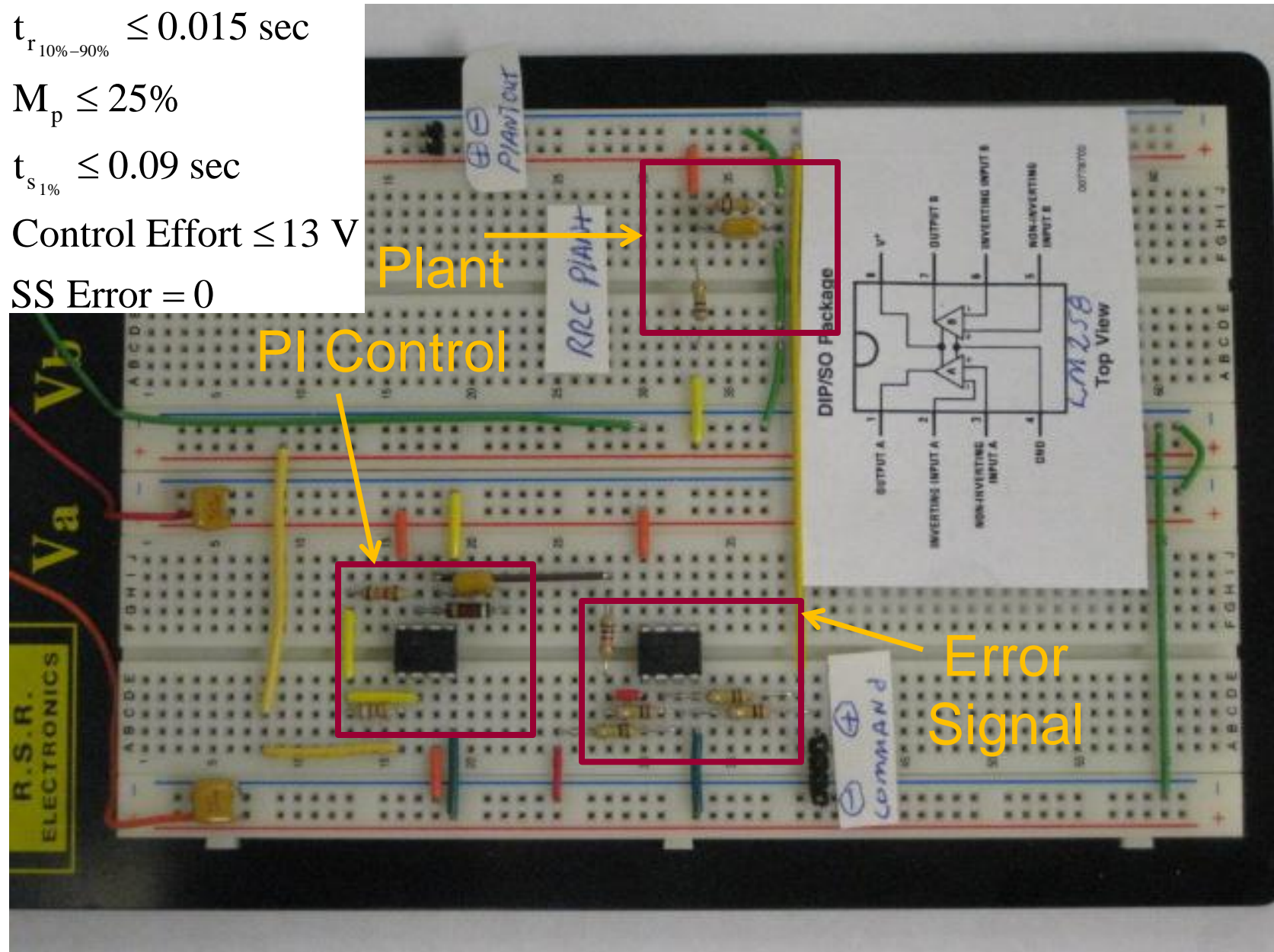
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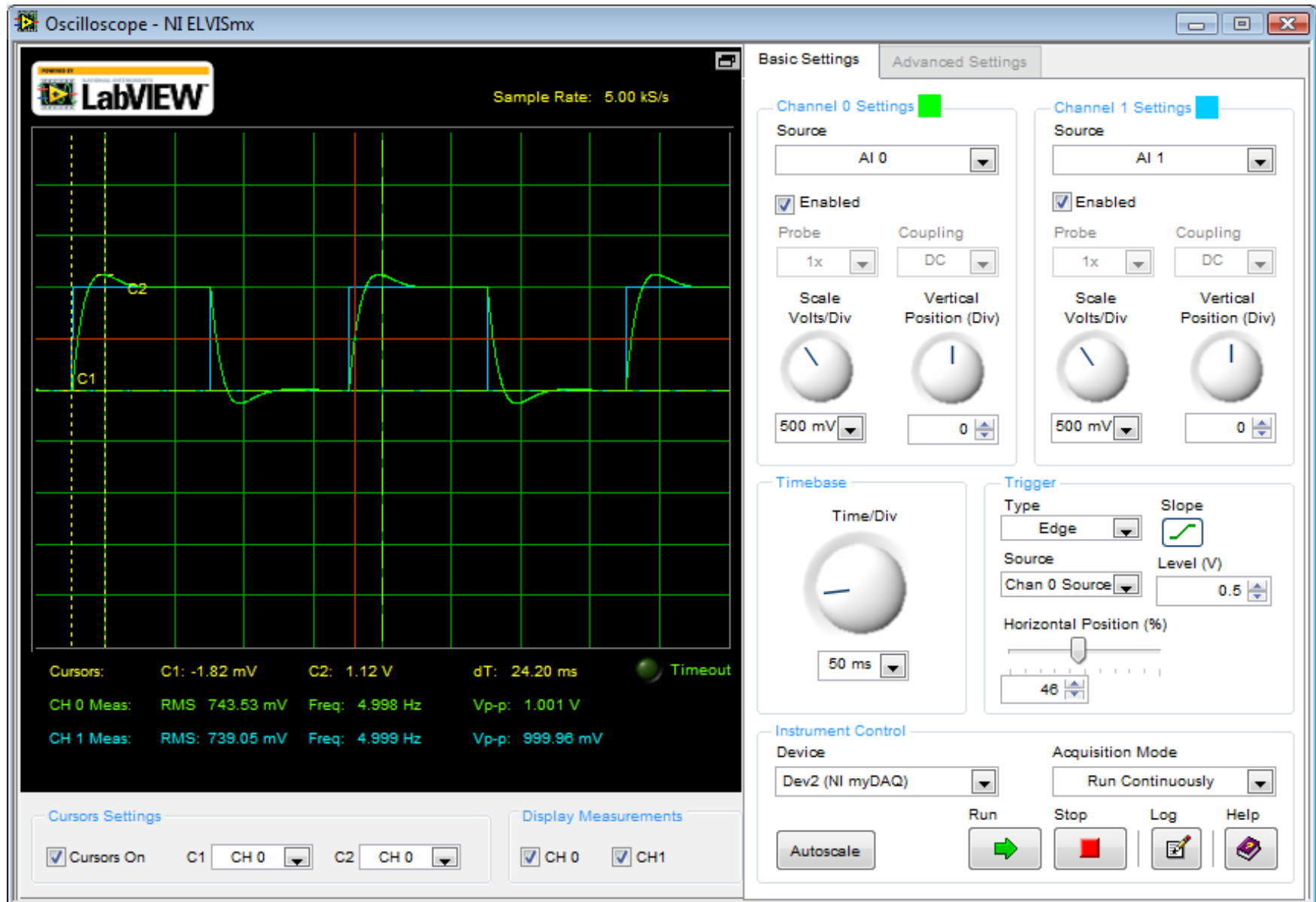
$$\text{Control Effort} \leq 13 \text{ V}$$

$$\text{SS Error} = 0$$



PI Analog Control of a 1st-Order Plant

Measurement: Closed-Loop Step-Response Plot – NI MyDAQ



Measurement: Closed-Loop Bode Plot – NI MyDAQ

