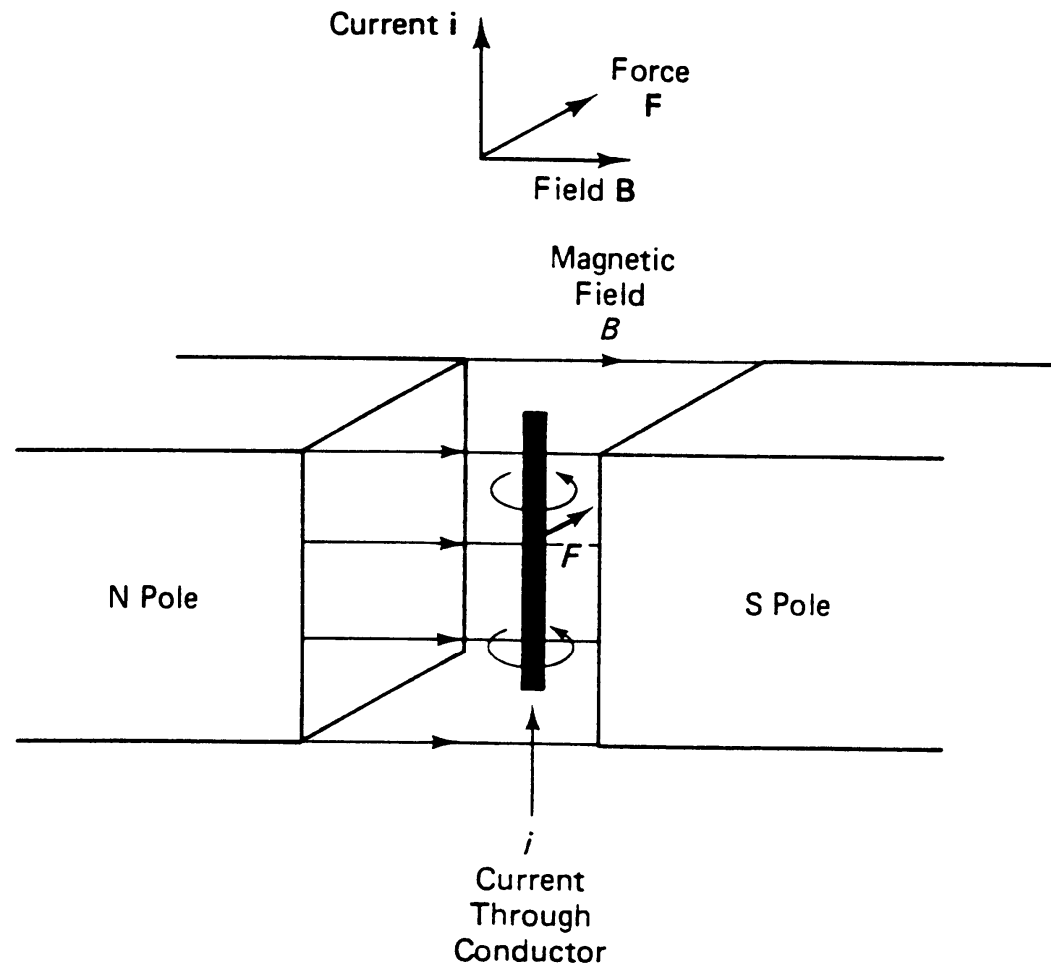


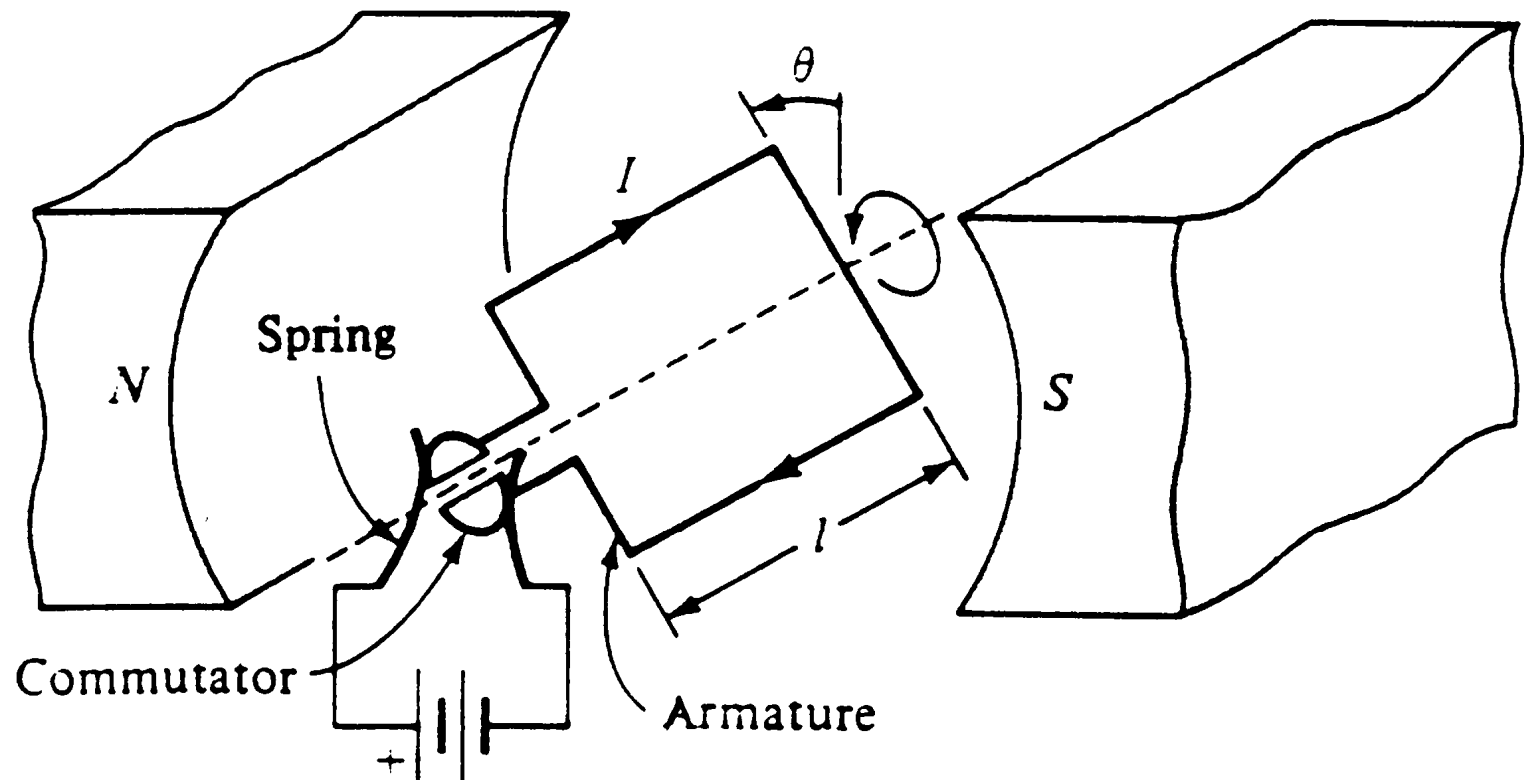
Elementary Approach to Permanent-Magnet Brushed DC Motor Modeling & Control

$$\vec{F} = \oint i d\vec{\ell} \times \vec{B} = B i \ell$$

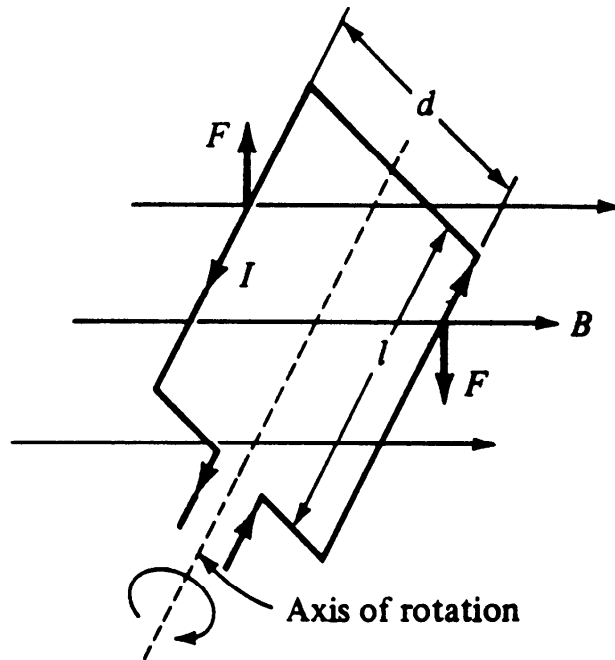
$$V_b = \int \vec{v} \times \vec{B} \cdot d\vec{\ell} = B \ell v$$



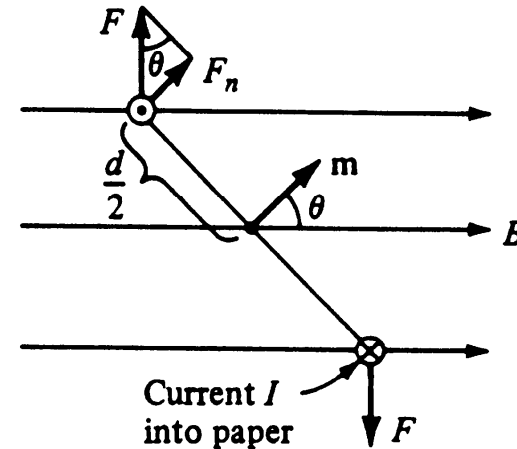
Elements of a Simple DC Motor



Torque of a DC Motor



(a)

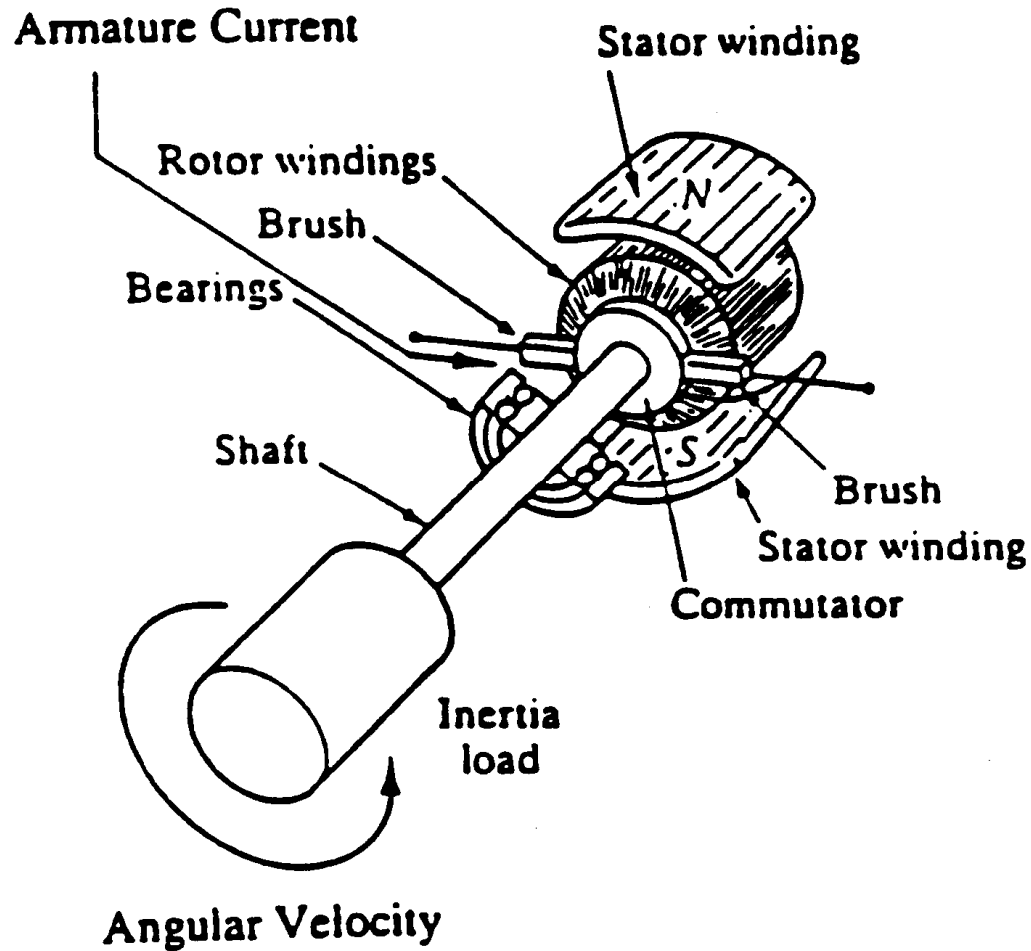


(b)

$$T = 2F_n \left(\frac{d}{2} \right) N = (iB\ell \sin \theta) dN = iABN \sin \theta = mBN \sin \theta$$

$$\vec{T} = N \left[\vec{m} \times \vec{B} \right]$$

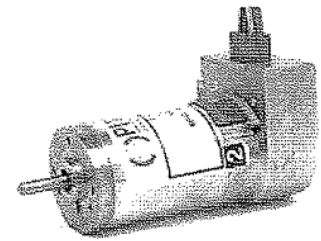
Schematic of a Brushed DC Motor



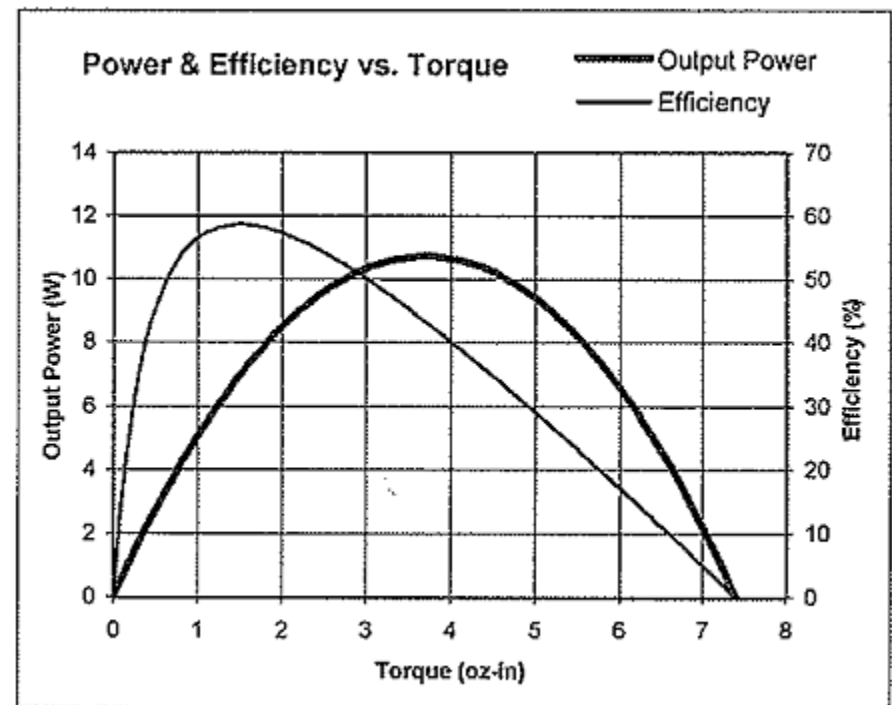
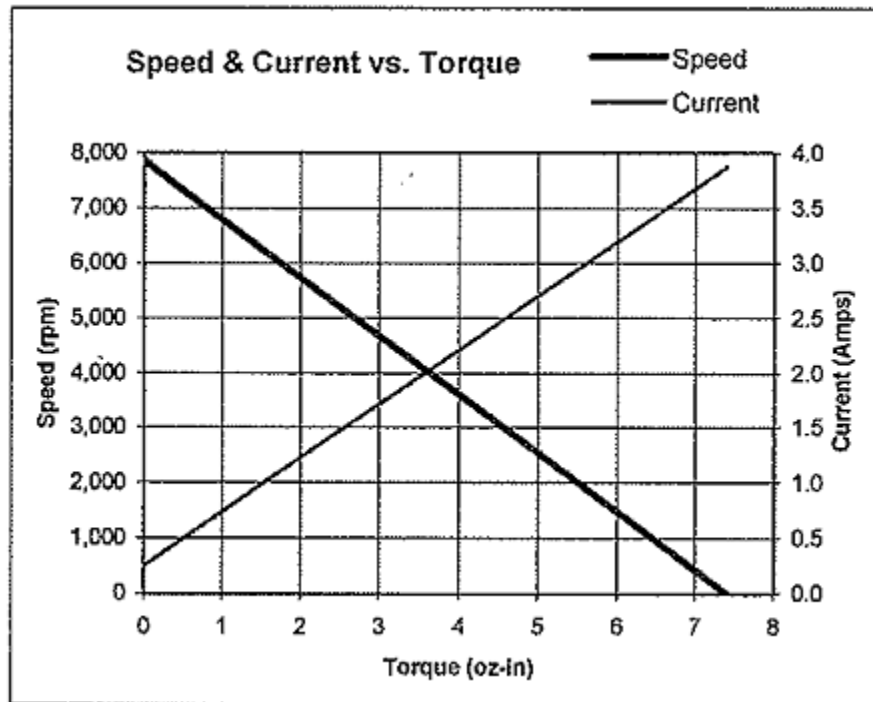
8322S001

Lo-Cog® DC Servo Motor

Pittman Brushed DC Motor



Assembly Data	Symbol	Units	Value	
Reference Voltage	E	V	12	
No-Load Speed	S_{NL}	rpm (rad/s)	7,847	(822)
Continuous Torque (Max.) ¹	T_C	oz-in (N-m)	1.6	(1.1E-02)
Peak Torque (Stall) ²	T_{PK}	oz-in (N-m)	7.4	(5.2E-02)
Weight	W_M	oz (g)	7.7	(218)
Motor Data				
Torque Constant	K_T	oz-in/A (N-m/A)	1.94	(1.37E-02)
Back-EMF Constant	K_E	V/krpm (V/rad/s)	1.43	(1.37E-02)
Resistance	R_T	Ω	3.10	
Inductance	L	mH	1.57	
No-Load Current	I_{NL}	A	0.25	
Peak Current (Stall) ²	I_P	A	3.88	
Motor Constant	K_M	oz-in/ \sqrt{W} (N-m/ \sqrt{W})	1.12	(7.91E-03)
Friction Torque	T_F	oz-in (N-m)	0.35	(2.5E-03)
Rotor Inertia	J_M	oz-in-s ² (kg-m ²)	1.4E-04	(9.9E-07)
Electrical Time Constant	τ_E	ms	0.52	
Mechanical Time Constant	τ_M	ms	15.6	
Viscous Damping	D	oz-in/krpm (N-m-s)	0.015	(1.0E-06)
Damping Constant	K_D	oz-in/krpm (N-m-s)	0.92	(6.2E-05)
Maximum Winding Temperature	θ_{MAX}	°F (°C)	311	(155)
Thermal Impedance	R_{TH}	°F/watt (°C/watt)	75.9	(24.4)
Thermal Time Constant	τ_{TH}	min	7.8	

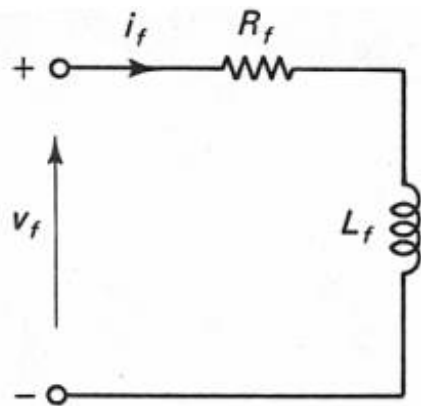


- Modeling Assumptions

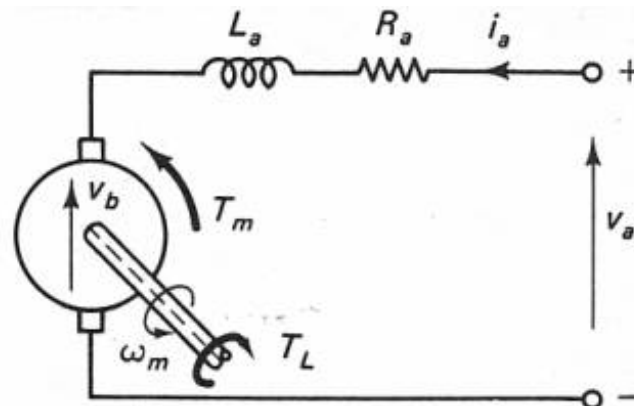
- The copper armature windings in the motor are treated as a resistance and inductance in series. The distributed inductance and resistance is lumped into two characteristic quantities, L and R .
- The commutation of the motor is neglected. The system is treated as a single electrical network which is continuously energized.
- The compliance of the shaft connecting the load to the motor is negligible. The shaft is treated as a rigid member. Similarly, the coupling between the tachometer and motor is also considered to be rigid.
- The total inertia J is a single lumped inertia, equal to the sum of the inertias of the rotor, the tachometer (if present), and the driven load.

- There exists motion only about the axis of rotation of the motor, i.e., a one-degree-of-freedom system.
- The parameters of the system are constant, i.e., they do not change over time.
- The damping in the mechanical system is modeled as viscous damping B . All stiction and dry friction are neglected, but can be included at a later time.
- Neglect noise on either the sensor or command signal.
- The amplifier dynamics are assumed to be fast relative to the motor dynamics. The unit is modeled by its DC gain, K_{amp} .
- The tachometer dynamics are assumed to be fast relative to the motor dynamics. The unit is modeled by its DC gain, K_{tach} .

Physical Modeling



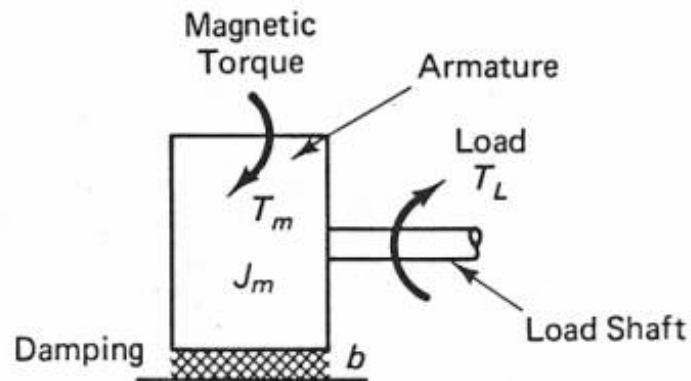
Stator (Field Circuit)



Rotor (Armature Circuit)

(a)

For a permanent-magnet DC motor,
 $i_f = \text{constant}$.



(b)

Mathematical Modeling

- The steps in mathematical modeling are as follows:
 - Define System, System Boundary, System Inputs and Outputs
 - Define Through and Across Variables
 - Write Physical Relations for Each Element
 - Write System Relations of Equilibrium and/or Compatibility
 - Combine System Relations and Physical Relations to Generate the Mathematical Model for the System

Physical Relations

$$V_L = L \frac{di_L}{dt}$$

$$V_R = Ri_R$$

$$T_B = B\omega$$

$$T_J = J\alpha = J\dot{\omega}$$

$$J \equiv J_{\text{motor}} + J_{\text{tachometer}} + J_{\text{load}}$$

$$T_m = K_t i_m$$

$$V_b = K_b \omega$$

$$P_{\text{out}} = T_m \omega = K_t i_m \omega$$

$$P_{\text{in}} = V_b i_m = K_b \omega i_m$$

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{K_t}{K_b}$$

$$P_{\text{out}} = P_{\text{in}}$$

$$K_t = K_b \equiv K_m$$

$$\left\{ \begin{array}{l} K_t (\text{oz} - \text{in} / \text{A}) = 1.3524 K_b (\text{V} / \text{krpm}) \\ K_t (\text{Nm} / \text{A}) = 9.5493 \times 10^{-3} K_b (\text{V} / \text{krpm}) \\ K_t (\text{Nm} / \text{A}) = K_b (\text{V} - \text{s} / \text{rad}) \end{array} \right.$$

System Relations + Equations of Motion

$$V_{\text{in}} - V_{\text{R}} - V_{\text{L}} - V_{\text{b}} = 0$$

$$T_{\text{m}} - T_{\text{B}} - T_{\text{J}} = 0$$

$$i_{\text{R}} = i_{\text{L}} = i_{\text{m}} \equiv i$$

$$V_{\text{in}} - Ri - L \frac{di}{dt} - K_{\text{b}} \omega = 0$$

$$J \dot{\omega} + B \omega - K_{\text{t}} i = 0$$

$$\begin{bmatrix} \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -B/J & K_{\text{t}}/J \\ -K_{\text{b}}/L & -R/L \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{\text{in}}$$

Steady-State Conditions

$$V_{\text{in}} - Ri - L \frac{di}{dt} - K_b \omega = 0$$

$$V_{\text{in}} - R \left(\frac{T}{K_t} \right) - K_b \omega = 0$$

$$T = \frac{K_t}{R} V_{\text{in}} - \frac{K_t K_b}{R} \omega$$

$$T_s = \frac{K_t}{R} V_{\text{in}} \quad \text{Stall Torque}$$

$$\omega_0 = \frac{V_{\text{in}}}{K_b} \quad \text{No-Load Speed}$$

Transfer Functions

$$V_{in} - Ri - L \frac{di}{dt} - K_b \omega = 0$$

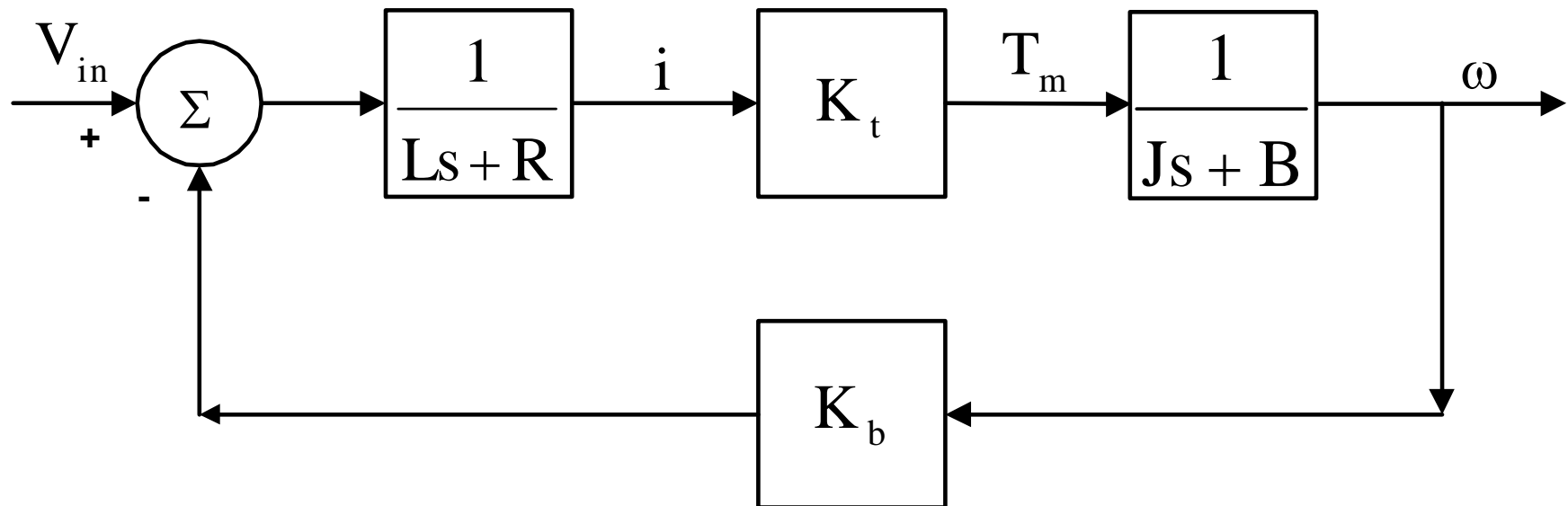
$$V_{in}(s) - (Ls + R)I(s) - K_b \Omega(s) = 0$$

$$J\dot{\omega} + B\omega - K_t i = 0$$

$$(Js + B)\Omega(s) - K_t I(s) = 0$$

$$\begin{aligned} \frac{\Omega(s)}{V_{in}(s)} &= \frac{K_t}{(Js + B)(Ls + R) + K_t K_b} = \frac{K_t}{JLs^2 + (BL + JR)s + (BR + K_t K_b)} \\ &= \frac{\frac{K_t}{JL}}{s^2 + \left(\frac{B}{J} + \frac{R}{L}\right)s + \left(\frac{BR}{JL} + \frac{K_t K_b}{JL}\right)} \end{aligned}$$

Block Diagram



Simplification

$$\tau_m = \frac{J}{B} \gg \tau_e = \frac{L}{R}$$

$$V_{in} - Ri - K_b \omega = 0$$

$$J\dot{\omega} + B\omega - K_t i = 0$$

$$J\dot{\omega} + B\omega = K_t i = K_t \left(\frac{1}{R} (V_{in} - K_b \omega) \right) = \frac{K_t}{R} (V_{in} - K_b \omega)$$

$$\dot{\omega} + \left(\frac{K_t K_b}{RJ} + \frac{B}{J} \right) \omega = \frac{K_t}{RJ} V_{in}$$

$$\dot{\omega} + \left(\frac{1}{\tau_{motor}} + \frac{1}{\tau_m} \right) \omega = \frac{K_t}{RJ} V_{in}$$

$$\dot{\omega} + \left(\frac{1}{\tau_{motor}} \right) \omega = \frac{K_t}{RJ} V_{in} \quad \text{since } \tau_m \gg \tau_{motor}$$

Control of DC Motors

- DC Motors can be operated over a wide range of speeds and torques and are particularly suited as variable-drive actuators.
- The function of a conventional servo system that uses a DC motor as the actuator is almost exclusively motion control (position and speed control). There are applications that require torque control and they usually require more sophisticated control techniques.
- Two methods of control of a DC motor are:
 - Armature Control
 - Field Control

- Armature Control

- Here the field current in the stator circuit is kept constant. The input voltage v_a to the rotor circuit is varied in order to achieve a desired performance. The motor torque can be kept constant simply by keeping the armature current constant because the field current is virtually constant in the case of armature control. Since v_a directly determines the motor back emf after allowance is made for the impedance drop due to resistance and inductance of the armature circuit, it follows that armature control is particularly suitable for speed manipulation over a wide range of speeds.

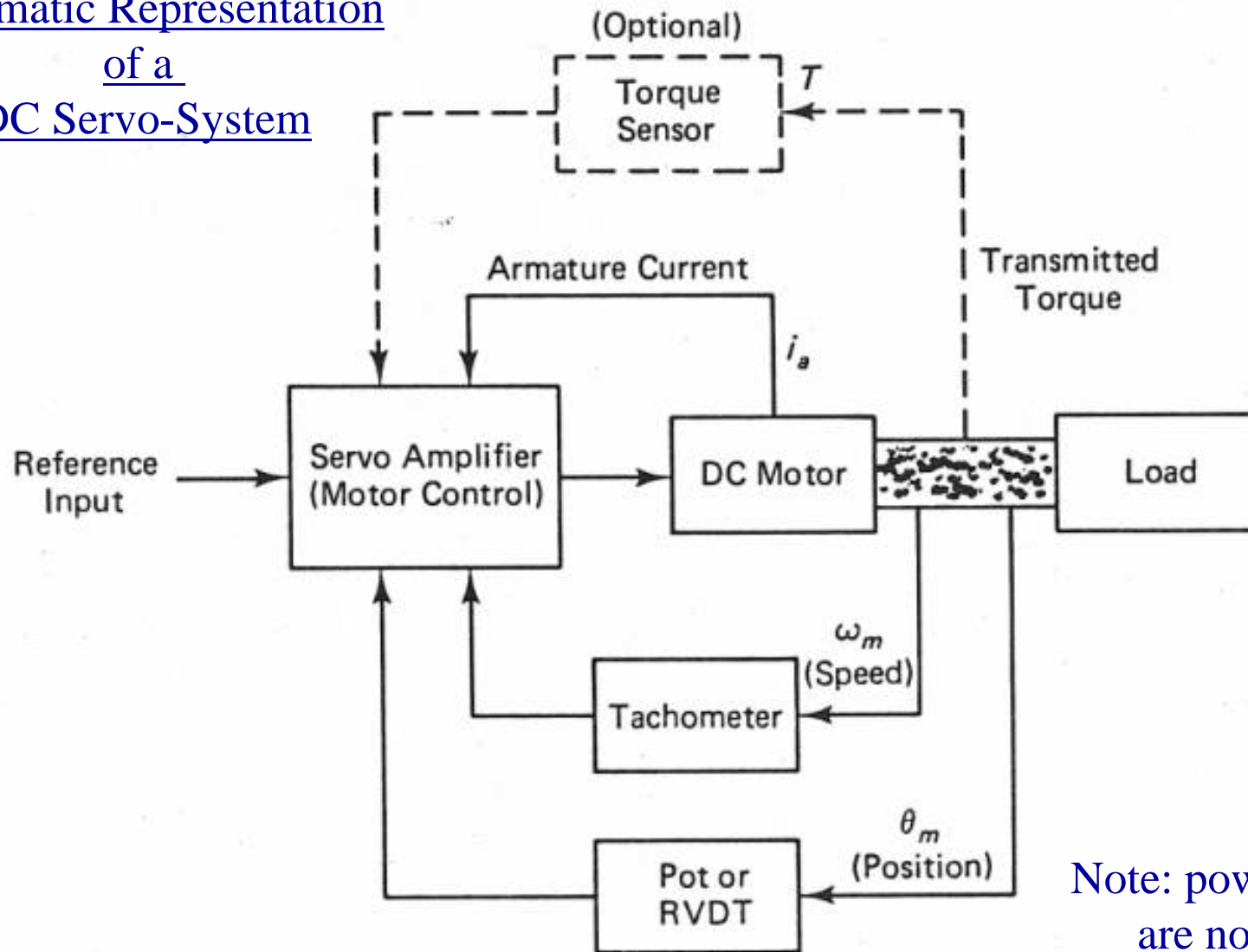
- Field Control

- Here the armature voltage (and current) is kept constant. The input voltage v_f to the field circuit is varied. Since i_a is kept more or less constant, the torque will vary in proportion to the field current i_f . Since the armature voltage is kept constant, the back emf will remain virtually unchanged. Hence the speed will be inversely proportional to i_f . Therefore, by increasing the field voltage, the motor torque can be increased while the motor speed is decreased, so that the output power will remain more or less constant in field control. Field control is particularly suitable for constant-power drives under varying torque-speed conditions, e.g., tape-transport mechanisms.

- DC Servomotors

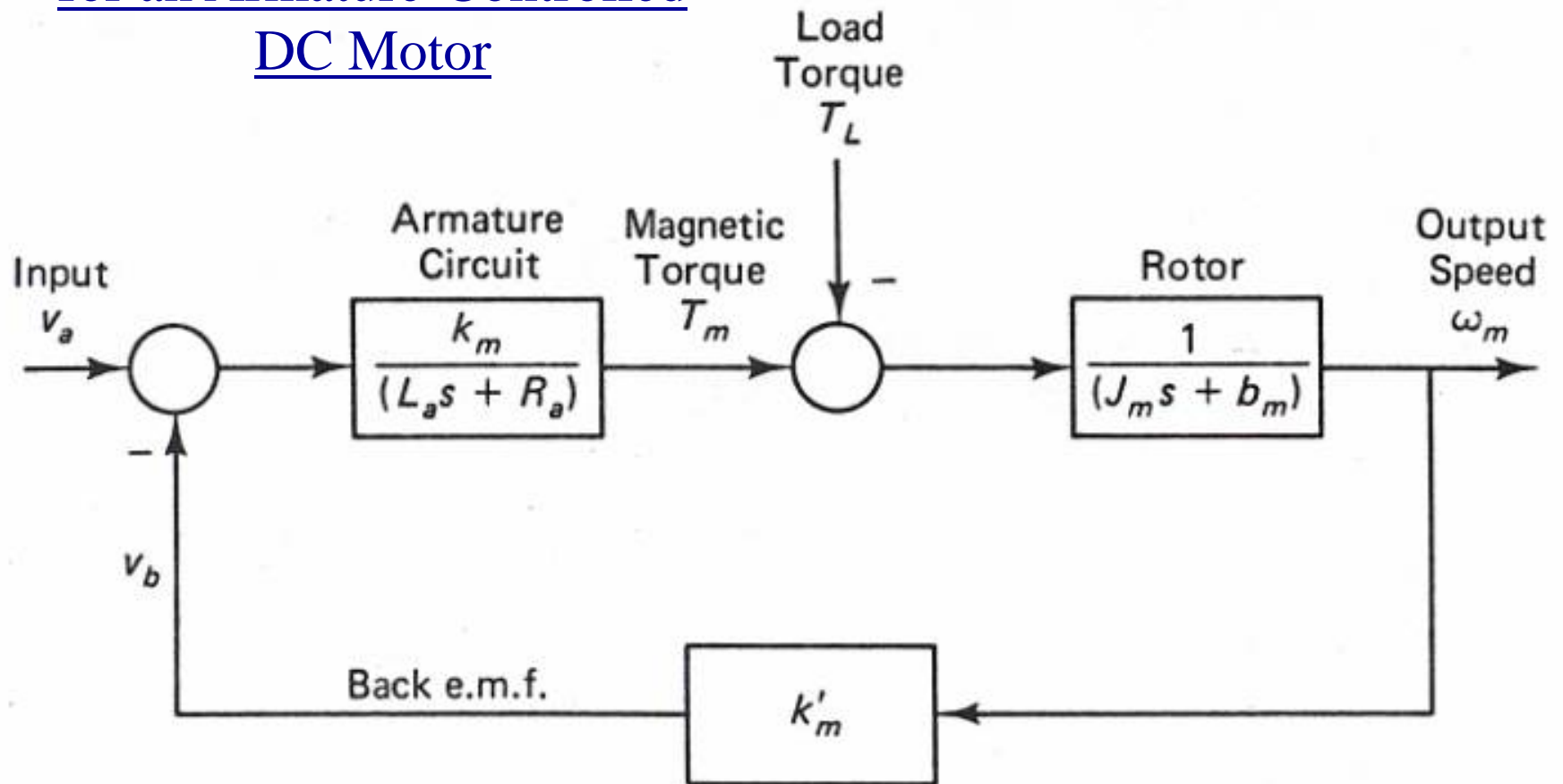
- DC servo-systems normally employ both velocity feedback and position feedback for accurate position control and also for accurate velocity control. Motion control requires indirect control of motor torque. In applications where torque itself is a primary output (e.g., metal-forming operations, tactile operations) and in situations where small motion errors could produce large unwanted forces (e.g., in parts assembly), direct control of motor torque would be necessary. For precise torque control, direct measurement of torque (e.g., strain gage sensors) would be required.

Schematic Representation of a DC Servo-System

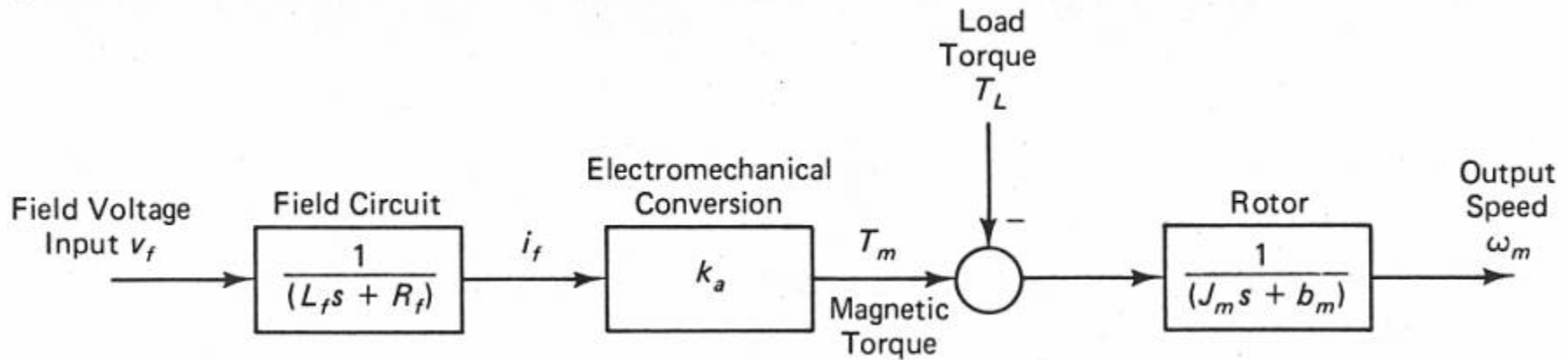


Note: power supplies
are not shown

Open-Loop Block Diagram for an Armature-Controlled DC Motor



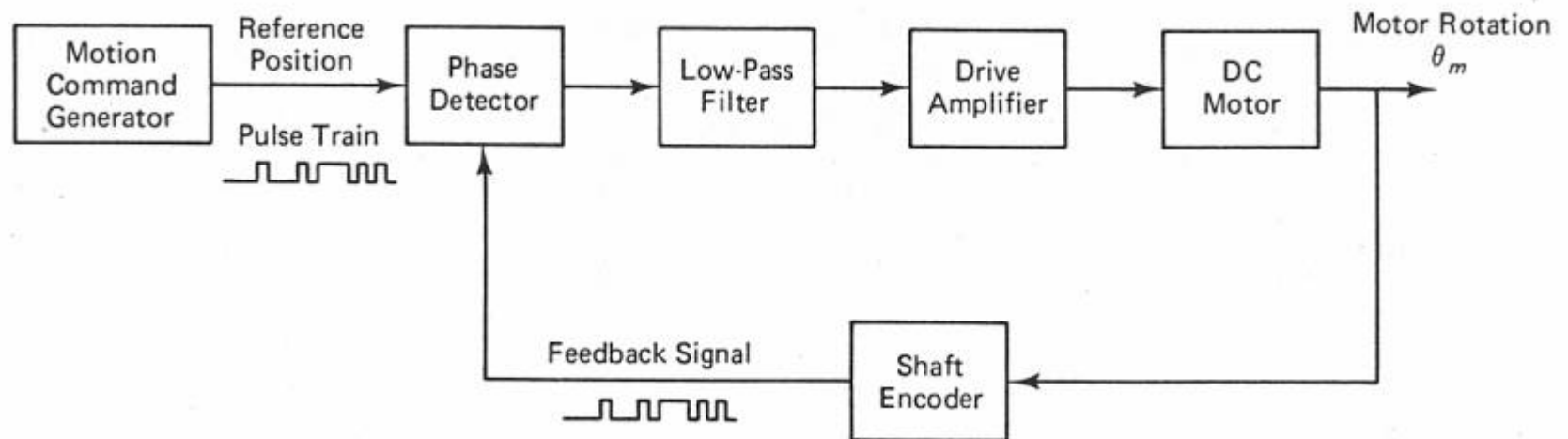
Open-Loop Block Diagram for a Field-Controlled DC Motor



- Phase-Locked Control

- Phase-locked control is a modern approach to controlling DC motors.
- This is a phase-control method. The objective is to maintain a fixed phase difference (ideally zero) between the reference signal and the position signal. Under these conditions, the two signals are phase-locked. Any deviation from the locked conditions will generate an error signal that will bring the motor motion back in phase with the reference command. In this manner, deviations due to external load changes on the motor are also corrected.
- How do you determine the phase difference between two pulse signals? One method is by detecting the edge transitions. An alternative method is to take the product of the two signals and then low-pass filter the result.

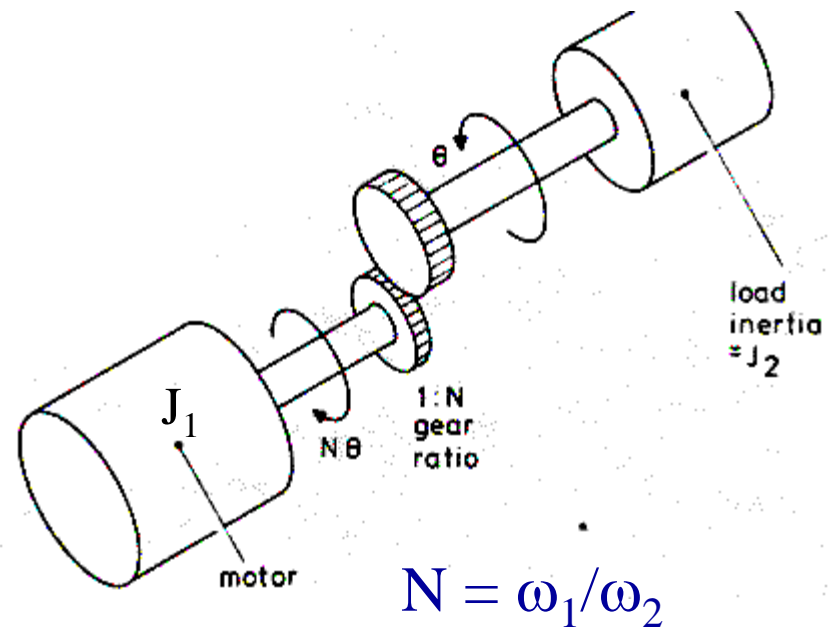
Schematic Diagram of a Phase-Locked Servo



Geared Systems, Optimum Gear Ratios, and Motor Selection

- Servomechanisms may be direct drive (motor coupled directly to load) or geared systems.
- For geared systems, choice of the motor also involves a choice of gear ratio.

Neglect backlash and elasticity in either gear teeth or shafts



- If we neglect frictional and other load effects, then the equation of motion for this system in terms of ω_2 is given by:

$$(J_2 + N^2 J_1) \frac{d\omega_2}{dt} = NT_m$$

- T_m is the electromagnetic torque from the motor.
- An important consequence of this result is that motor inertia is often the dominant inertia in a servo system. Consider the following numerical example:
 - Motor rotor inertia $J_1 = 1$, load inertia $J_2 = 100$, and gear ratio $N = 100$.
 - Physically the load appears 100 times “larger” than the motor, but because of the high (but not unusual) gear ratio, the motor’s inertia effect is $N^2 J_1$ or 10,000, i.e., 100 times larger than the load.
 - Thus measures to “lighten” the load inertia are misplaced; we should really be striving for a lower inertia motor.

- When frictional and other load effects are negligible and inertia is dominant, an optimum gear ratio that maximizes load shaft acceleration for a given input torque exists and may be found as follows:

$$(J_2 + N^2 J_1) \frac{d\omega_2}{dt} = NT_m$$

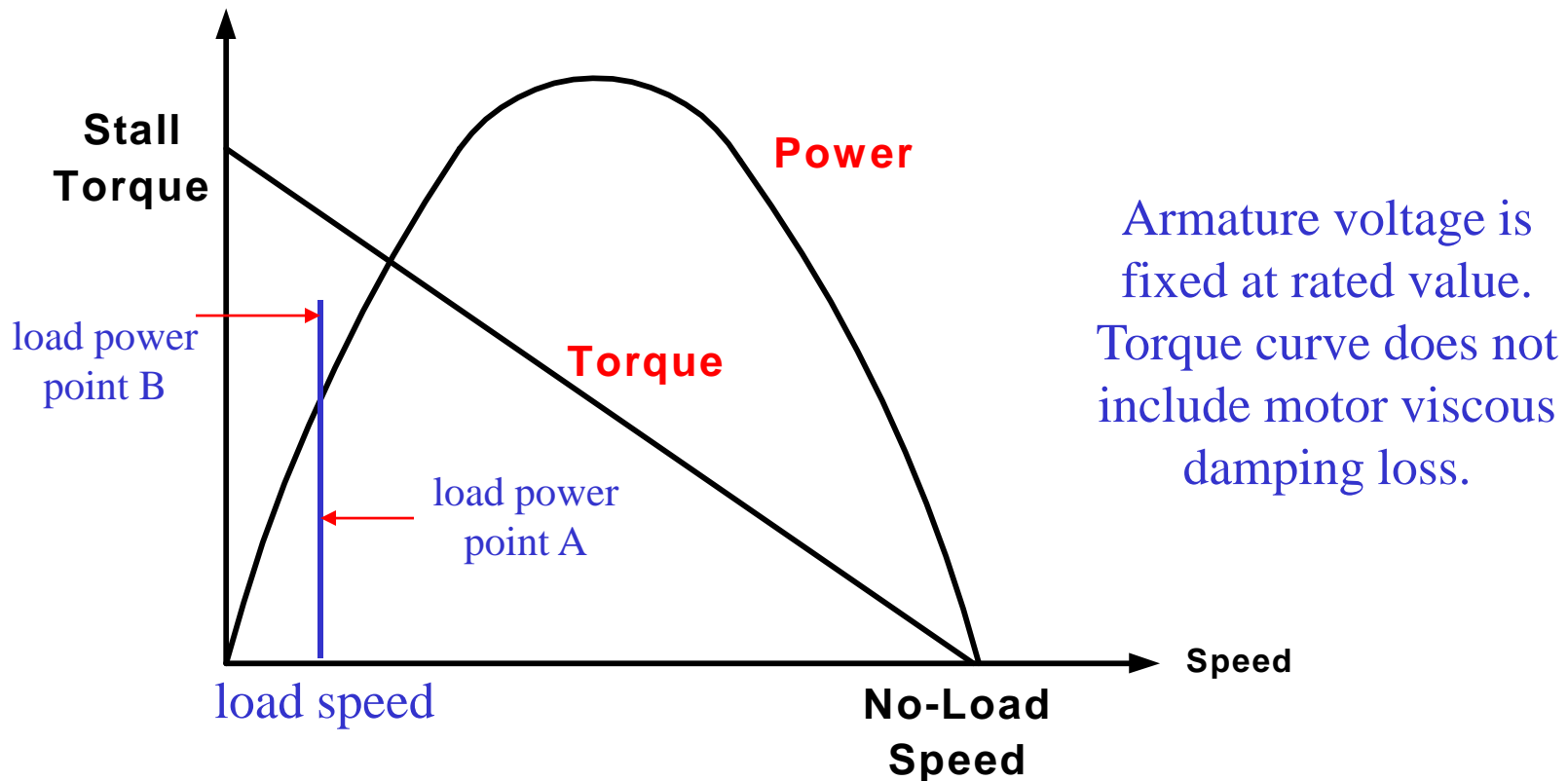
$$\frac{d\omega_2}{dt} = \frac{NT_m}{J_2 + N^2 J_1}$$

$$\frac{d}{dN} \left(\frac{d\omega_2}{dt} \right) = \frac{(J_2 + N^2 J_1) T_m - NT_m (2NJ_1)}{(J_2 + N^2 J_1)^2} = 0$$

$$N_{\text{opt}} = \sqrt{\frac{J_2}{J_1}}$$

Referred inertia of the motor rotor is equal to the actual load inertia

- When friction, load, and acceleration torques are all significant, gear ratio selection is less straightforward.
- Consider the linear speed/torque curve (typical of an armature-controlled dc motor with fixed field). Power is the product of speed and torque, hence the parabolic speed/power curve.



- One possible design criterion requires that we operate the motor at the maximum power point (which occurs at one half the no-load speed) when it is supplying the greatest demand of the load.
- One could compute the maximum power demand at maximum velocity; however, it is unlikely that this maximum speed will coincide with the peak power speed of the motor; thus a direct drive would not satisfy our requirement to operate at the peak power point.
- The figure shows two load power points at the same maximum load speed.
- If maximum load power corresponded to point A, a direct drive would be possible since the motor has more power than needed at this speed. The excess power simply means that more acceleration is available than is required.

- If instead our calculations had given point B, direct drive would not be possible since the load requires more power than the motor can supply at that speed. However, this load is less than the peak motor power, so suitable gearing can reconcile the supply/demand mismatch.
- Since one finds both direct-drive and geared systems in practical use, it is clear that the one scheme is not always preferable to the other.
- Direct drive is favored because of reduced backlash and compliance and longer motor life due to lower speed.
- The advantages of geared drives include smoother motor operation at higher speeds, possibly higher torsional natural frequency, and the lower cost of smaller motors. Of course, if a sufficiently large motor is simply not available, then a geared drive is a necessity.

Motor Selection Considerations

- Torque and speed are the two primary considerations in choosing a motor for a particular application. Motor manufacturers' data that are usually available to users include the following specifications:
- Mechanical Specifications
 - Mechanical time constant
 - Speed at rated load
 - Rated torque
 - Frictional torque
 - Dimensions and weight
 - No-load speed

- No-load acceleration
 - Rated output power
 - Damping constant
 - Armature moment of inertia
- Electrical Specifications
 - Electrical time constant
 - Armature resistance and inductance
 - Compatible drive circuit specifications (voltage, current, etc.)
 - Input power
 - Field resistance and inductance
- General Specifications
 - Brush life and motor life

- Heat transfer characteristics
- Coupling methods
- Efficiency
- Mounting configuration
- Operating temperature and other environmental conditions

- It should be emphasized that there is no infallible guide to selecting the best motor. There are always several workable configurations. Constraints (e.g., space, positioning resolution) can often eliminate several designs.
- The engineer must make the motor drive system work both electrically and mechanically. Look at the motor-to-load interface before looking at the electrical drive-to-motor interface.
- In a typical motion control application the requirement will be to overcome some load frictional force and move a mass through a certain distance in a specified time.

- The designer should weigh the following:
 - Moment of Inertia
 - Torque
 - Power
 - Cost
- Load Inertia
 - For optimum system performance, the load moment of inertia should be similar to the motor inertia.
 - When gear reducers intervene between motor and load, the reflected load inertia is J_L/N^2 , where N is the gear ratio.
 - If the motor inertia J_M is equal to the reflected load inertia, the fastest load acceleration will be achieved, or conversely, the torque to obtain a given acceleration will be minimized.

- Therefore, matched inertias are best for fast positioning.
- Peak power requirements are minimized by selecting the motor inertia so that the reflected load inertia is 2.5 times as large as the motor inertia. The torque will be increased but the maximum speed will be further reduced. A load inertia greater than 2.5 times the motor inertia is less than ideal, but it should not present any problems if the ratio is less than 5. A larger motor inertia implies that the same performance can be achieved at a lower cost by selecting a smaller motor.
- There is a wide range of motor inertias on the market today. Overlap is extensive. An engineer can virtually consider any type of motor including brushless and stepper, at the first stage of the design-inertia match.

- Torque

- The motor must supply sufficient torque T_m to overcome the load friction and to accelerate the load over a distance (radians) s in time τ .
- Torque and acceleration at the motor are given by:

$$\left. \begin{array}{l} \alpha_m = N\alpha_L \\ T_L = T_f + J_L\alpha_L \end{array} \right\} T_m = \frac{T_L}{N} + J_m\alpha_m \Rightarrow T_m = \frac{1}{N} \left[T_f + \alpha_L (J_L + N^2 J_m) \right]$$

- For linear acceleration over distance s in time τ : $\alpha_L = \frac{2s}{\tau^2}$
- For a damped ($\zeta = 0.7$) second-order response over distance s : $(\alpha_L)_{\max} = \omega_n^2 s$

- Allowances should always be made for variations in load and bearing behavior as well as motor production variations.
- An initial design should be planned without a gear reducer. In many cases direct drive is not possible because load torque requirements far exceed the torque delivered by a motor of reasonable size.
- Critical needs on space or weight can lead to gear reducers for otherwise perfectly matched motor/load systems.
- The problem with gear reducers is gear backlash. If gears mesh too tightly, there is severe sliding friction between teeth which can cause lockup. Thus, the teeth spacing, backlash, is a tradeoff between reducing the power loss within the gears (loose fit) or improving position accuracy (tight fit) of the load.

- Power

- Besides maximum torque requirements, torque must be delivered over the load speed range. The product of torque and speed is power. Total power P is the sum of the power to overcome friction P_f and the power to accelerate the load P_a , the latter usually the dominant component:

$$P = P_f + P_a = T_f \omega + J \alpha \omega$$

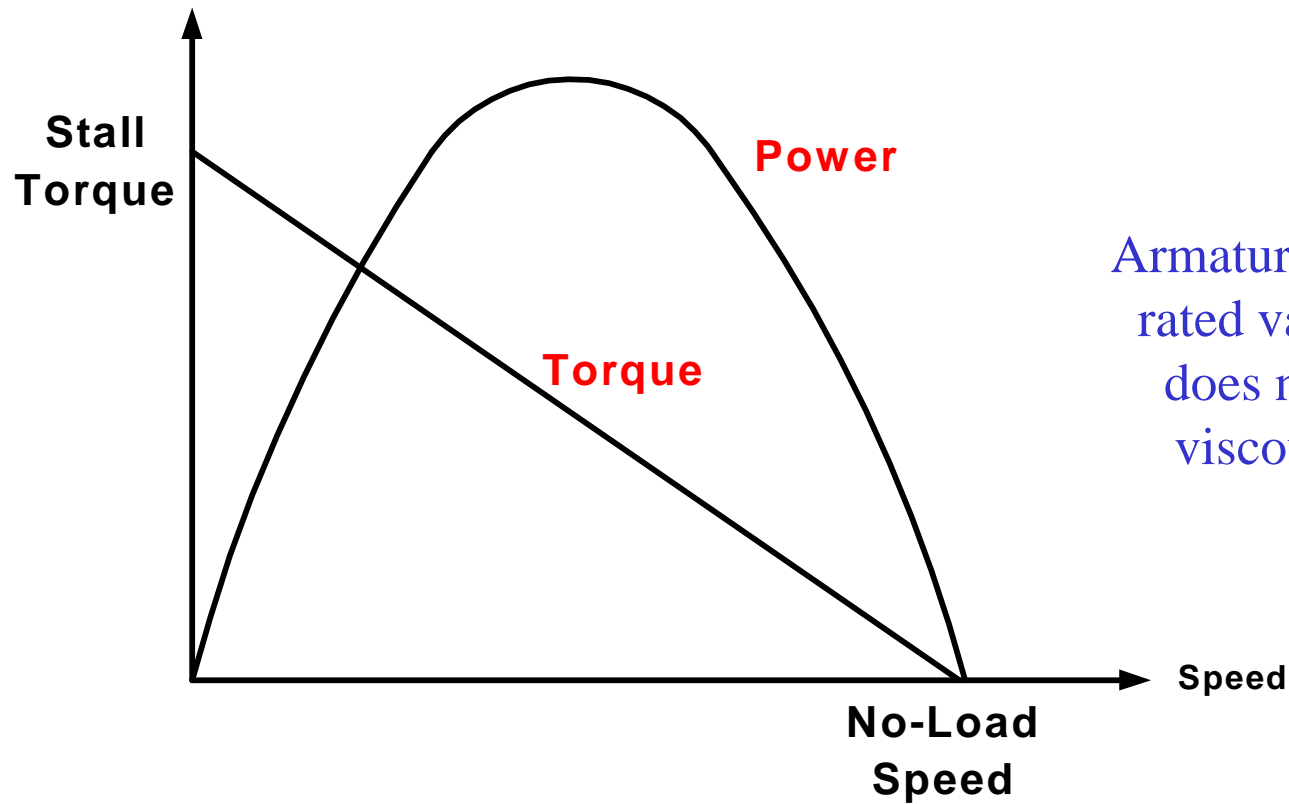
- Peak power required during acceleration depends upon the velocity profile. If the load is linearly accelerated over distance s in time τ , the maximum power is:

$$(P_a)_{\max} = \frac{4Js^2}{\tau^3}$$

- If the load undergoes a damped ($\zeta = 0.7$) second-order response over distance s , the maximum power is:

$$(P_a)_{\max} = (0.146)J\omega_n^3 s^2$$

- It is interesting to note that to accelerate a load in one half the time will require eight times the power.
- The torque-speed curve for permanent-magnet DC motors is a linear line from stall torque to no-load speed. Therefore, the maximum power produced by the motor is the curve midpoint or one-fourth the stall torque and maximum speed product.



Armature voltage is fixed at rated value. Torque curve does not include motor viscous damping loss.

- The maximum speed for a permanent-magnet DC motor is 10,000 RPM, while for a brushless DC motor it is $> 20,000$ RPM.
 - As a starting point, choose a motor with double the calculated power requirement.
- Cost
 - Among several designs the single most important criterion is cost.
 - Although it may be more prudent to choose the first workable design when only several units are involved, high-volume applications demand careful study of the economic tradeoffs.

- For example, permanent-magnet DC motors operate closed loop and the cost of an encoder can equal if not exceed the motor cost. In addition, stepper and brushless motors have electronic expenses greater than brushed motor electronic expenses.