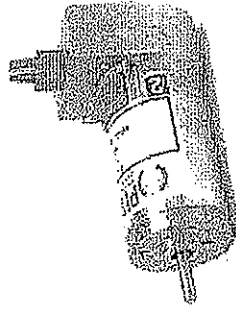
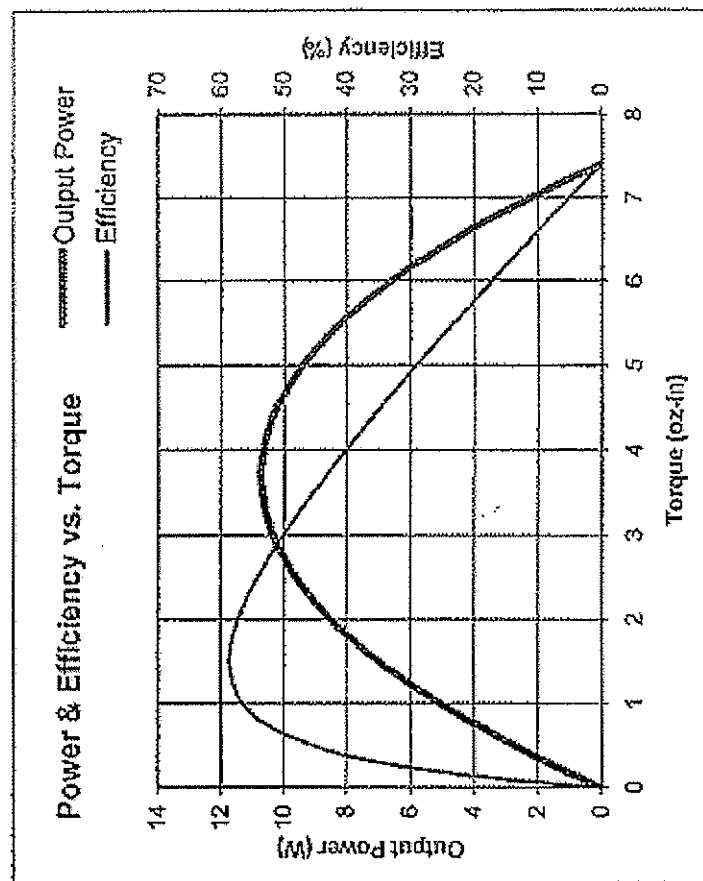
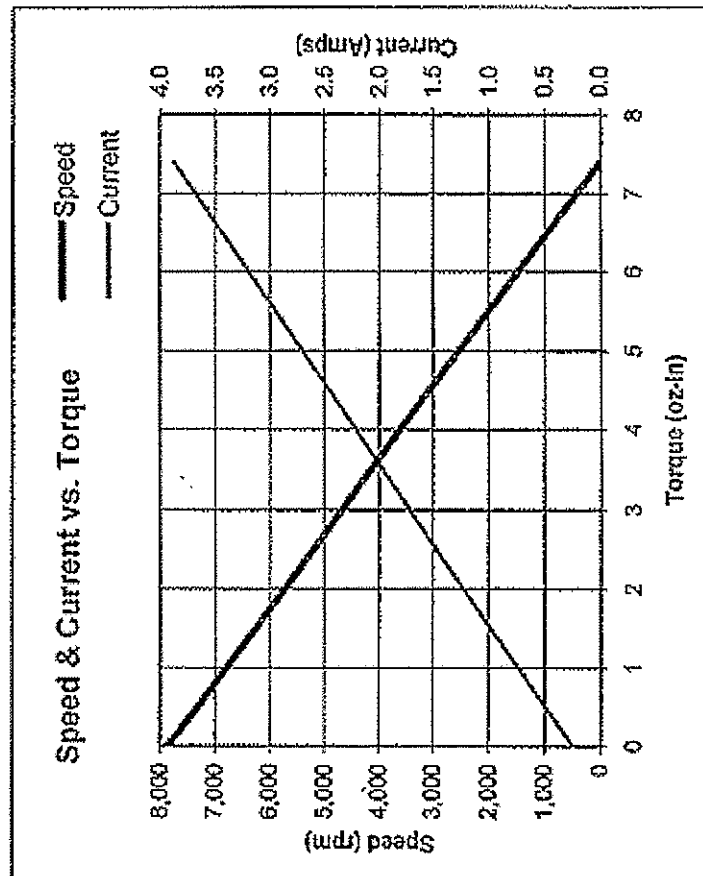


8322S001

Lo-Cog® DC Servo Motor

Pittman Brushed DC Motor

Assembly Data		Symbol	Units	Value
Reference Voltage		E	V	12
No-Load Speed		S_{NL}	rpm (rad/s)	7,847 (822)
Continuous Torque (Max.) ¹		T_C	oz-in (N-m)	1.6 (1.1E-02)
Peak Torque (Stall) ²		T_{PK}	oz-in (N-m)	7.4 (5.2E-02)
Weight		W_M	oz (g)	7.7 (218)
Motor Data				
Torque Constant		K_T	oz-in/A (N-m/A)	1.94 (1.37E-02)
Back-EMF Constant		K_E	V/krpm (V/rad/s)	1.43 (1.37E-02)
Resistance		R_T	Ω	3.10
Inductance		L	mH	1.57
No-Load Current		I_{NL}	A	0.25
Peak Current (Stall) ²		I_P	A	3.88
Motor Constant		K_M	oz-in/ \sqrt{W} (N-m/ \sqrt{W})	1.12 (7.91E-03)
Friction Torque		T_F	oz-in (N-m)	0.35 (2.5E-03)
Rotor Inertia		J_M	oz-in-s ² (kg-m ²)	1.4E-04 (9.9E-07)
Electrical Time Constant		τ_e	ms	0.52
Mechanical Time Constant		τ_M	ms	15.6
Viscous Damping		D	oz-in/krpm (N-m-s)	0.015 (1.0E-06)
Damping Constant		K_D	oz-in/krpm (N-m-s)	0.92 (6.2E-05)
Maximum Winding Temperature		θ_{MAX}	°F (°C)	311 (155)
Thermal Impedance		R_{TH}	°F/watt (°C/watt)	75.9 (24.4)
Thermal Time Constant		τ_{TH}	min	7.8



Pittmann Brushed DC Motor Modeling

1

K. Craig

Equations of Motion

$$e_{in} = L \frac{di}{dt} + Ri + K_b \omega$$

$$J \dot{\omega} + B\omega = K_t i \quad (\text{Coulomb friction neglected})$$

Open-Loop Transfer Function $\omega/e_{in}(s)$:

$$\frac{\omega}{e_{in}} = \frac{K_t/JL}{s^2 + \left(\frac{B}{J} + \frac{R}{L}\right)s + \left(\frac{BR}{JL} + \frac{K_t K_b}{JL}\right)}$$

$$\left. \begin{aligned} \tau_m &= J/B \\ \tau_e &= L/R \end{aligned} \right\} \tau_m \gg \tau_e$$

$$J \dot{\omega} + B\omega = K_t i = K_t \left[\frac{1}{R} (e_{in} - K_b \omega) \right] = \frac{K_t}{R} (e_{in} - K_b \omega)$$

$$\dot{\omega} + \left(\frac{K_t K_b}{RJ} + \frac{B}{J} \right) \omega = \frac{K_t}{RJ} e_{in}$$

$$\dot{\omega} + \left(\frac{1}{\tau_{motor}} + \frac{1}{\tau_m} \right) \omega = \frac{K_t}{RJ} e_{in} \Rightarrow \dot{\omega} + \left(\frac{1}{\tau_{motor}} \right) \omega = \frac{K_t}{RJ} e_{in} \quad \tau_m \gg \tau_{motor}$$

$$\tau_{motor} = \frac{RJ}{K_t K_b}$$

Analysis:

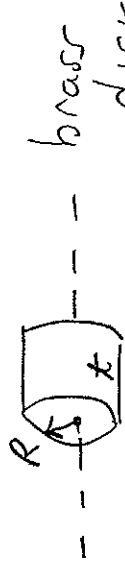
Pittman 83225001 Brushed DC Servo Motor

2

$$J_{\text{motor}} = 9.9 \times 10^{-7} \text{ kg-m}^2$$

$$J_{\text{load}} = \frac{1}{2} m R^2$$

$$m = (\pi R^2)(t)(\rho_{\text{brass}})$$



$$= 9.93 \times 10^{-6} \text{ kg-m}^2$$

$$\left\{ \begin{array}{l} \rho_{\text{brass}} = 8500 \frac{\text{kg}}{\text{m}^3} \\ t = .25'' \\ = .00635 \text{ m} \\ R = .01875 \text{ m} \end{array} \right.$$

$$\frac{J_L}{J_m} = 10.03$$

$$J_{\text{total}} = 10.92 \times 10^{-6} \text{ kg-m}^2$$

$$B = 1.0 \times 10^{-6} \text{ N-m-s}$$

$$K_t = 1.37 \times 10^{-2} \text{ (N-m)/A}$$

$$K_b = 1.37 \times 10^{-2} \text{ V/rad/s}$$

$$R = 3.10 \Omega$$

$$L = 1.57 \times 10^{-3} \text{ H}$$

$$\tau_{\text{motor}} \dot{\omega} + \omega = \frac{1}{K_b} e_{\text{in}}$$

$$\left\{ \begin{array}{l} \tau_m = J/B = 10.92 \\ \tau_e = L/R = 5.06 \times 10^{-4} \end{array} \right\} \tau_m \gg \tau_e$$

$$\frac{\tau_m}{\tau_e} = 2.16 \times 10^4$$

$$\left\{ \begin{array}{l} \tau_{\text{motor}} = \frac{RJ}{K_t K_b} = 0.180 \end{array} \right.$$

$$\tau_m / \tau_{\text{motor}} = 60.5$$

$$\tau_{\text{motor}} \dot{\omega} + \omega = \frac{1}{K_b} e_{in}$$

$$(0.180) \dot{\omega} + \omega = (73.0) e_{in}$$

$$\tau \dot{\omega} + \omega = K e_{in} \quad 1^{\text{st}} - \text{Order ODE}$$

$$\frac{\omega}{e_{in}} = \frac{K}{\tau D + 1} \quad \begin{cases} \tau = 0.180 \text{ sec} \\ K = 73.0 \end{cases}$$

verify
by
experiment.

From Pittman Data Sheet

$$\text{Motor Constant } K_M = 7.91 \times 10^{-3}$$

$$= \frac{K_t}{\sqrt{R}}$$

$$W = \frac{N-m}{\sqrt{W}} \quad W = \text{watts} \quad (I^2 R)$$

$$\tau_f = 2.5 \times 10^{-3} \text{ N-m Friction Torque (Coulomb friction dynamic)}$$

$$\tau_e = 0.52 \times 10^{-3} \text{ sec} = \frac{L}{R} \quad \text{same as } \tau_e$$

$$\tau_M = 15.6 \times 10^{-3} \text{ sec} = \frac{R J}{K_t K_b} \quad \text{same as } \tau_{\text{motor}}$$

(note: $J = J_m$ on data sheet)

$$K_0 = \text{damping constant} = \frac{K_t K_b}{R} = 6.2 \times 10^{-5} \text{ N-m-s}$$

(same units as B)

Steady-State Analysis

4

$$E_{in} = L \frac{d\lambda}{dt} + R_L + K_b \omega \Rightarrow E_{in} = R_L + K_b \omega$$

$$T \dot{\omega} + B \omega = K_t \lambda \Rightarrow B \omega = K_t \lambda$$

Define $T = B \omega + T_f + T_L$ (includes all load torques)

$$E_{in} - R_L - K_b \omega = 0$$

$$E_{in} - R \left(\frac{T}{K_t} \right) - K_b \omega = 0 \Rightarrow T = \frac{K_t}{R} E_{in} - \frac{K_t K_b}{R} \omega$$

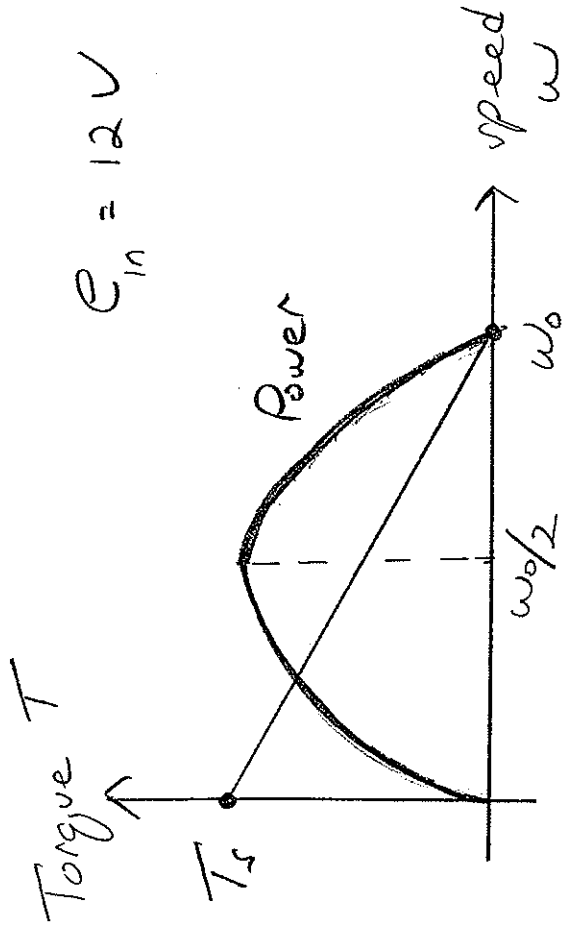
$$= K_M \sqrt{R} E_{in} - K_D \omega \quad (\text{Pittman parameters})$$

$$T = \frac{K_t}{R} E_{in} - \frac{K_t K_b}{R} \omega$$

Linear Torque-Speed Relation

$$\omega = 0 \Rightarrow T_s = \frac{K_t}{R} E_{in} \quad \text{stall torque}$$

$$T = 0 \Rightarrow \omega_0 = E_{in} / K_b \quad \text{no-load speed}$$



$$E_{in} = 12 \text{ V} \quad \left\{ \begin{array}{l} T_s = \frac{k_t E_{in}}{R} = 0.053 \text{ N-m} \\ \omega_0 = E_{in}/k_b = 8.76 \text{ rad/s} \end{array} \right.$$

(822 in data sheet)

$$\begin{aligned} \text{Power } P = T\omega &= \omega \left[\frac{k_t E_{in}}{R} - \frac{k_t k_b}{R} \omega \right] \\ &= \omega \left[T_s - \frac{T_s}{\omega_0} \omega \right] = \omega T_s \left[1 - \frac{\omega}{\omega_0} \right] \\ P &= T_s \left[\omega - \frac{\omega^2}{\omega_0} \right] \end{aligned}$$

$$\frac{dP}{d\omega} = T_s - 2 \frac{T_s}{\omega_0} \omega = 0 \Rightarrow \omega_{\max} = \frac{\omega_0}{2}$$

$$\text{Peak Current (stall)} = E_{in}/R = 3.87 \text{ A}$$

$$\text{No-load Current} = 0.25 \text{ A}$$

Pittman Data Sheet:

$$\text{no-load speed} = 822 \text{ rad/s}$$

$$\text{no-load current} = 0.25 \text{ A}$$

Interpret this information

no-load current is a measure of friction losses.

$$\text{no-load current} = 0.25 \text{ A}$$

$$E_m = L \frac{d\omega}{dt} + R_L + K_b \omega \quad \frac{d\omega}{dt} = 0$$

$$\text{no-load speed } \omega = \frac{E_m - R_L}{K_b} = \frac{12 - (3.10)(0.25)}{(1.37 \times 10^{-2})} = 819.3 \text{ rad/s}$$

(Close to 822 given)

At the no-load speed

$$T_m = K_t L = (1.37 \times 10^{-2})(0.25) = 3.43 \times 10^{-3} \text{ N-m}$$

at steady-state

$$T_m = T_{\text{viscous}} + T_{\text{coulomb}}$$

$$= B\omega + T_f$$

$$= (1.0 \times 10^{-6})(819.3) + (2.5 \times 10^{-3}) = 3.32 \times 10^{-3} \text{ N-m}$$

Load Torque vs. Efficiency

7

Rated Armature Voltage = 12 V

Load Torque $T_L = 1.5 \text{ oz-in} = (1.502 \times 10^{-10}) \left(\frac{1 \text{ N}}{3.602} \right) \left(\frac{1 \text{ m}}{39.37} \right) = 1.06 \times 10^{-2} \text{ N-m}$

$B = 1.0 \times 10^{-6} \text{ N-m-s}$

$K_t = 1.37 \times 10^{-2} \frac{\text{N-m}}{\text{A}}$

$T_f = 2.5 \times 10^{-3} \text{ N-m}$

} During Steady State

$K_t \omega = B\omega + T_f + T_L$

$\omega = \frac{1}{B} (K_t \omega - T_f - T_L)$

Also during steady state:

$e = R\omega + K_b \omega$

$\omega = \frac{e}{R} - \frac{K_b \omega}{R} = \frac{e}{R} - \frac{K_b}{RB} (K_t \omega - T_f - T_L)$

Solve for ω : $\omega = \frac{e + \left(\frac{K_b}{B} \right) (T_f + T_L)}{R + \frac{K_b K_t}{B}}$

Substitute numbers:

$\omega = \frac{1}{190.8} \left[12 + (1.37 \times 10^{-2}) (2.5 \times 10^{-3} + T_L) \right] = 1.004 \text{ A}$

$\omega = \frac{1}{1.0 \times 10^{-6}} \left[(1.37 \times 10^{-2}) (\omega) - (2.5 \times 10^{-3}) - T_L \right] = 654.8 \text{ rad/s}$

substitution

$$\text{Power Input: } P_{in} = e I = (12)(1.004) = 12.05 \text{ W}$$

$$\text{Power Output: } P_{out} = T_L \omega = (1.06 \times 10^{-2})(654.8) = 6.94 \text{ W}$$

$$\text{Efficiency \%} = \frac{P_{out}}{P_{in}} (100) = \frac{6.94}{12.05} (100) = 57.6 \%$$

Matcher

Portman Graphy

$$\text{Losses: } P_{LR} = R_L^2 = (3.10)(1.004)^2 = 3.125 \text{ W}$$

$$P_{friction} = (B\omega + T_F)\omega = [(1.0 \times 10^{-6})(654.8) + (2.5 \times 10^{-3})] \times (654.8)$$

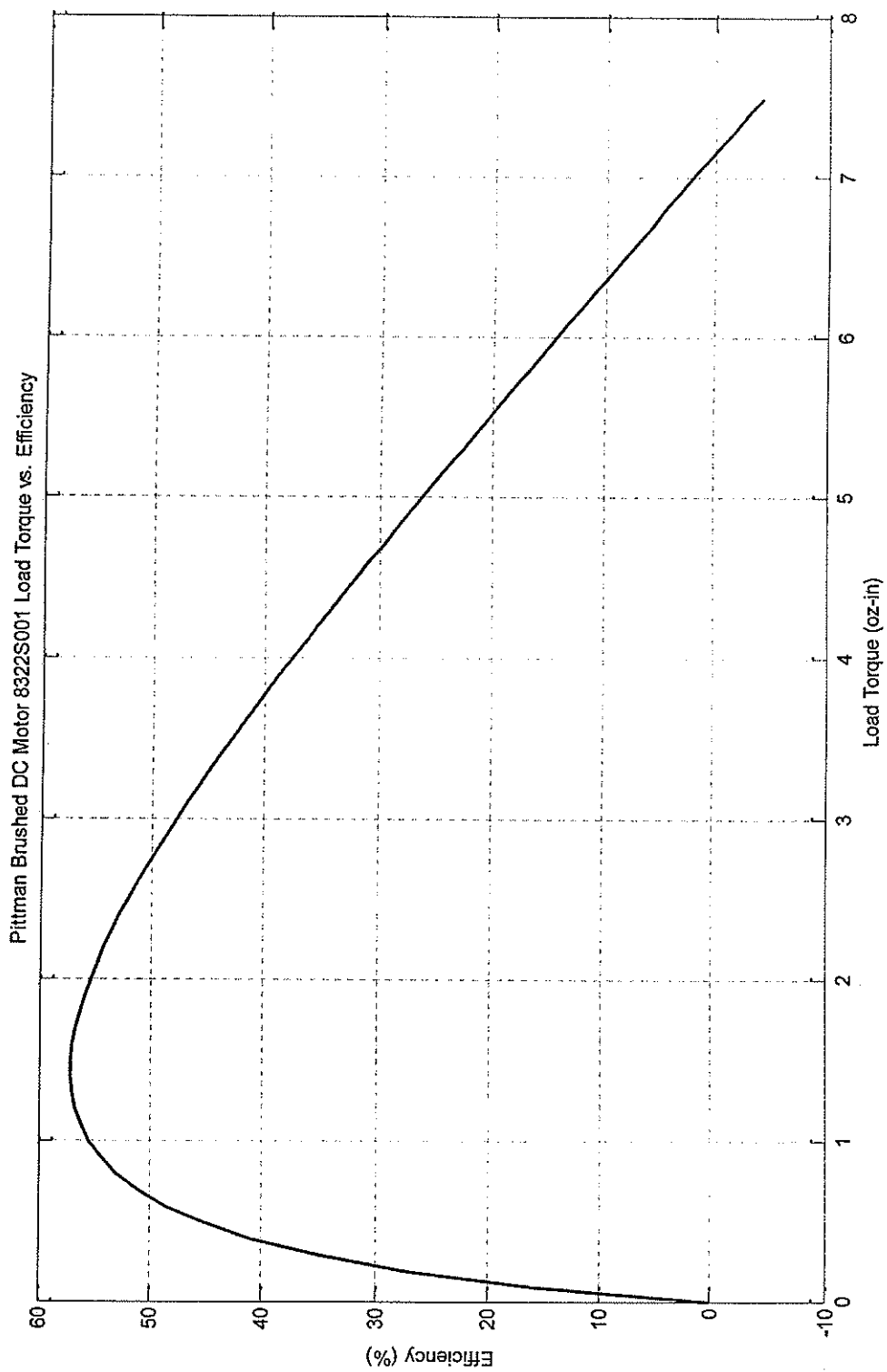
$$= 2.066 \text{ W}$$

$$\text{Total Losses: } 2.066 + 3.125 = 5.191 \text{ W}$$

$$\text{Power Input} = \text{Power Output} + P_{losses}$$

$$12.05 = 6.94 + 5.19$$

$$12.05 \approx 12.13 \quad \checkmark$$



% Pittman Motor 8322S001 Load Torque vs. Efficiency Plot

```

B = 1.0E-6;
Kt = 1.37E-2;
Tf = 2.5E-3;
Kb = 1.37E-2;
R = 3.10;
TL = 0:1:7.5;
TL = TL/3.6/39.37;
e = 12;
i = (1/(R + Kb*Kt/B))*(e + (Kb/B)*(Tf + TL));
omega = (1/B)*((Kt*i) - Tf - TL);
Pin = e*i;
Pout = TL.*omega;
E = (Pout./Pin)*100;
plot(TL*3.6*39.37,E)

```

MatLab M-File