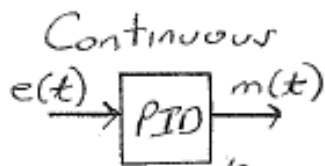


PID Computer Control

K. Craig



$$m(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

Differential Equation

Differential Operator: $D = \frac{d}{dt}$
 $D = s$ s Laplace Transform $\frac{1}{D} = \int dt$

Transfer Function

$$\frac{M}{E}(D) = K_p + \frac{K_I}{D} + K_D D$$



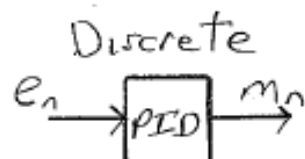
Summary PID

Continuous: K_p K_I K_D

Diff Eq. \rightarrow D Transfer function
 \rightarrow s Transfer function

Discrete: K_0 K_1 K_2

Difference Eq. \rightarrow B Transfer function
 \rightarrow z Transfer function



$$m_n = m_{n-1} + K_0 e_n + K_1 e_{n-1} + K_2 e_{n-2}$$

Difference Equation

$$\begin{cases} K_0 = K_p + \frac{K_D}{T_s} \\ K_1 = -K_p + K_I T_s - \frac{2K_D}{T_s} \\ K_2 = \frac{K_D}{T_s} \end{cases} \quad T_s = \text{sample period}$$

\rightarrow with forward integration approximation + zero-order hold in front.

Backshift Operator: $B e_n = e_{n-1}$

$B = 1/z$ z-Transform $B^2 e_n = e_{n-2}$

$$m_n = B m_n + K_0 e_n + K_1 B e_n + K_2 B^2 e_n$$

$$\frac{m_n}{e_n} = \frac{K_0 + K_1 B + K_2 B^2}{1 - B} = \frac{K_0 z^2 + K_1 z + K_2}{z(z-1)}$$

Transfer function