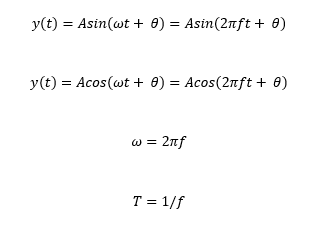
**Sinusoidal Functions and Simple Harmonic Motion**

Many physical processes and reactions can be modeled using sinusoidal functions or by combining sinusoidal and exponential functions.

1. **Sinusoidal Functions**

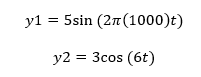
A sinusoidal function is a periodic function that oscillates continually at some frequency. Some equations describing a sinusoidal function and useful relationships are:



|  |  |  |
| --- | --- | --- |
| A | = | Amplitude (units depend on application) |
| f | = | Frequency (Hz) |
| ω | = | Angular frequency (rad/s) |
| θ | = | Phase shift (rad) |
| T | = | Period or time for one cycle (s) |

A sinusoidal function with a frequency of 10 Hz would complete 10 cycles or periods every second.

Consider the following two sinusoidal functions:



1. Answer each of the following questions and be sure to include units in your answer.

* What is the period of y1? \_\_\_\_\_\_\_\_\_\_
* What is the period of y2? \_\_\_\_\_\_\_\_\_\_
* How much time does it take for y1 to complete 3 cycles? \_\_\_\_\_\_\_\_\_\_\_\_
* How much time does it take for y2 to complete 3 cycles? \_\_\_\_\_\_\_\_\_\_\_\_

1. Using the subplot command, plot exactly three cycles of y1 and y2 in separate sub-windows of the same plot window with y1 on the top and y2 on the bottom. Be sure to label and title each graph appropriately. Then paste your MATLAB commands and graph in the space below.

**MATLAB COMMANDS:**

**GRAPH:**

1. **Modeling Simple Harmonic Motion**

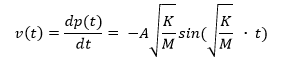
Sinusoids are used to model simple harmonic motion as illustrated in the figure below that shows a mass attached to a spring. If we assume no loss of energy, the displacement of the mass looks like a cosine wave. In this lab, we will assume the oscillations are undamped.

Assuming simple harmonic motion, the displacement of the mass is given by:

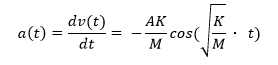


|  |  |  |
| --- | --- | --- |
| A | = | Initial Displacement of Mass (how much spring is compressed initially) |
| K | = | Spring Constant (N/m) |
| M | = | Mass of suspended mass and spring (kg) |

The velocity of the mass is the derivative of displacement with respect to time:



The acceleration of the mass is the derivative of velocity with respect to time:



The spring constant is a measure of the resistance of the spring to being compressed or stretched. According to Hooke’s Law, the force necessary to compress or stretch a spring by an amount, x, is F = Kx. So, a spring constant of 200 N/m simply means that it would take a force of 2N to stretch (or compress) the spring by 0.01m and it would take a force of 4N to stretch (or compress) the spring by 0.02m.

1. Open a script file by clicking on New Script. On the first line of the script, type the following: % Lab 4: Simple Harmonic Motion. Remember starting a line with a % turns it into a comment line – not something that executes. On the second line of the script file, put a comment line (start with %) with your name, your recitation section, and today’s date. Then save your script file as Lab4\_*yourlastname*.

***Note: filenames follow the exact same rules as variable names in MATLAB – they must start with a letter which can be followed by any combination of letters, numbers, and underscores. You cannot leave spaces in filenames or insert any other type of characters.***

**Put all of the MATLAB commands that you use to complete the rest of this lab in your script file.**

1. Assume the spring constant, K, is 200 N/m, the combined mass of the spring and suspended mass, M, is 0.2 kg and the spring is initially compressed by 3cm then released. Calculate the frequency (in Hz) and the period of the oscillations.

**Frequency = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Hz**

**Period = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ seconds**

1. Use the subplot command to plot approximately four cycles of the displacement, velocity, and acceleration of the mass. Displacement should be the top plot, velocity should be the middle plot, and acceleration should be the bottom plot. Use a time increment of 0.001 seconds. Label and title your graph appropriately then paste the graph in the space below.

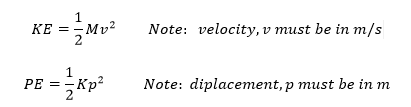
**GRAPH:**

1. Fill in the table below using your graph and the data cursor tool.

**Note: If the acceleration or the velocity is changing from positive to negative or from negative to positive on either side of the data point you are trying to locate on the graph then just enter a value of 0 in the table.**

|  |  |  |
| --- | --- | --- |
| **Position of Mass** | **Velocity (cm/s)** | **Acceleration (cm/s2)** |
| **0 cm with mass moving down** |  |  |
| **0 cm with mass moving up** |  |  |
| **+ 3 cm (spring maximally compressed)** |  |  |
| **3 cm (spring maximally stretched)** |  |  |
| **+2 cm with mass moving down** |  |  |

1. Use your graph to determine how far the mass has traveled in 0.65 seconds. **Hint: in one complete cycle, the mass travels 12 cm (i.e., from + 3 cm to – 3 cm and back to + 3 cm).**
2. Using your values from the previous table, calculate the kinetic energy and potential energy of the mass at equilibrium and when the spring is maximally stretched/compressed:



|  |  |  |  |
| --- | --- | --- | --- |
| **Position of Mass** | **Kinetic Energy (J)** | **Potential Energy (J)** | **Total Energy (J)** |
| **0 cm (equilibrium position)** |  |  |  |
| **+ 3 cm (spring maximally compressed)** |  |  |  |
| **+ 2 cm** |  |  |  |

1. Using your graphs and calculations, describe in words what is happening to the mass in terms of velocity, acceleration, kinetic energy, and potential energy as it is passing through the equilibrium point (displacement of 0 cm).
2. Using your graphs and calculations, describe in words what is happening to the mass in terms of velocity, acceleration, kinetic energy, and potential energy when the spring is maximally compressed.
3. In your script file, type the command: ***figure*** after your current sent of MATLAB commands. This will create a new figure window so your old plot doesn’t go away.
4. Use the subplot command to subdivide the figure into 3 sub-windows then do the following:

* In the top plot, plot the displacement for a spring constant, K, of 200 N/m, a total mass, M, of 0.2 kg, and an initial compression of 3 cm (i.e., same thing you already plotted once before).
* In the middle plot, using exactly the same time range as your first plot, plot the displacement for a spring constant, K, of 800 N/m, a total mass, M, of 0.2 Kg, and an initial compression of 3 cm.
* In the bottom plot, using exactly the same time range as your first plot, plot the displacement for a spring constant, K of 200 N/m, a total mass, M, of 0.8 kg, and an initial compression of 3 cm.
* Paste your plot in the space below and if you haven’t already done so, put your MATLAB commands in the script file.

**GRAPH:**

1. How does quadrupling the spring constant (much stiffer spring) effect the frequency of the oscillations? Be specific.
2. How does quadrupling the mass effect the frequency of the oscillations? Be specific.