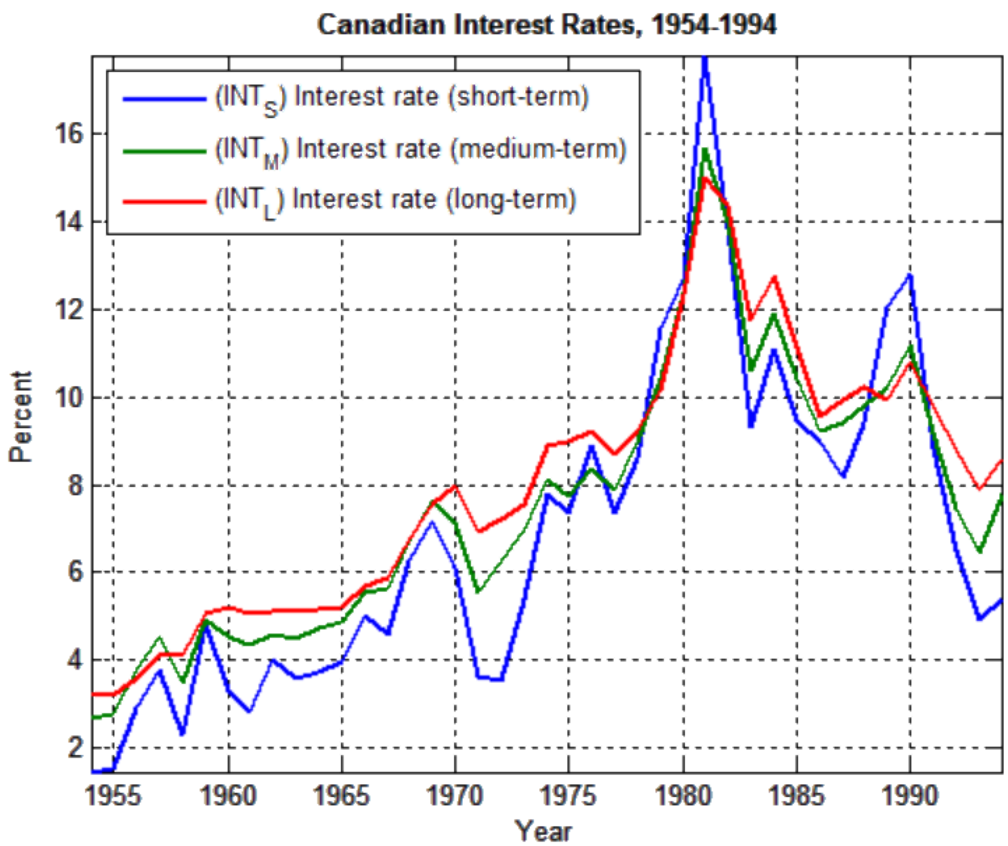
Cointegration

William Mueller
Econometrics Toolbox

Common Stochastic Trends



Reference:

MacKinnon, J. G. "Numerical Distribution Functions for Unit Root and Cointegration Tests." Journal of Applied Econometrics. v. 11, 1996, pp. 601-618.

Terminology

A univariate time series y_t is **integrated** if it can be brought to stationarity through differencing. The number of differences required to achieve stationarity is called the **order of integration**. Time series of order d are denoted $I(d)$. Stationary series are denoted $I(0)$.

An n-dimensional time series $\boldsymbol{y}_t$ is **cointegrated** if some linear combination $\beta_1 y_{1t} + \ldots + \beta_n y_{nt}$ of the component variables is stationary. The combination is called a **cointegrating relation**, and the coefficients $\beta = (\beta_1, \ldots , \beta_n)'$ form a **cointegrating vector**.

Engle-Granger Test for Cointegration

Data: `Y` (numObs-by-numVars)

Cointegrating regression:

```
Y(:,1) = Y(:,2:end)*b + X*a + e
```

Test: $\hat{e}$ for a unit root

Test statistics do not have the usual Dickey-Fuller or Phillips-Perron distributions!

The Cointegrated VAR Model

$$\Delta \mathbf{y}_t = AB'\mathbf{y}_{t-1} + \sum_{i=1}^{q} B_i \Delta \mathbf{y}_{t-i} + D\mathbf{x} + \boldsymbol{\varepsilon}_t$$

adjustment speeds

deterministic terms

cointegrating vectors

VAR in differences

Johansen Test for Cointegration

Comprehensive maximum likelihood **framework** that supports cointegration analysis, model estimation, and testing of restrictions on cointegrating relations and adjustment speeds.

5 models of cointegration:

Case	Form of $AB'\mathbf{y}_{t-1} + Dx$	Model Interpretation
H2	$AB'\mathbf{y}_{t-1}$	There are no intercepts or trends in the cointegrating relations and there are no trends in the data. This model is only appropriate if all series have zero mean.
H1*	$A(B\mathbf{y}_{t-1} + c_0)$	There are intercepts in the cointegrating relations and there are no trends in the data. This model is appropriate for nontrending data with nonzero mean.
H1	$A(B'\mathbf{y}_{t-1}+c_0) + c_1$	There are intercepts in the cointegrating relations and there are linear trends in the data. This is a model of *deterministic cointegration*, where the cointegrating relations eliminate both stochastic and deterministic trends in the data.
H*	$A(B'\mathbf{y}_{t-1} + c_0 + d_0t) + c_1$	There are intercepts and linear trends in the cointegrating relations and there are linear trends in the data. This is a model of *stochastic cointegration*, where the cointegrating relations eliminate stochastic but not deterministic trends in the data.
H	$A(B'\mathbf{y}_{t-1} + c_0 + d_0t) + c_1 + d_1t$	There are intercepts and linear trends in the cointegrating relations and there are quadratic trends in the data. Unless quadratic trends are actually present in the data, this model may produce good in-sample fits but poor out-of-sample forecasts.

Testing Cointegrating Vectors

`B` is n-by-r (numVars-by-numCIRelations)